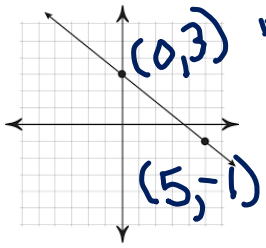
2.1 Linear & Quadratic Functions and Modeling

Target 2A: Graph and Solve Quadratic Functions

Review of Prior Concepts

Find the slope of the line:

a)



$m = \frac{-1-3}{5-0} = \boxed{\frac{-4}{5}}$

b) that contains the points $(2,-3)$ and $(5,1)$

$m = \frac{1--3}{5-2} = \boxed{\frac{4}{3}}$

More Practice

Finding Slope of a Line

http://www.coolmath.com/algebra/08-lines/06-finding-slope-line-given-two-points-01
https://www.khanacademy.org/math/algebra/two-var-linear-equations/slope/v/slope-of-a-line
http://www.mathwarehouse.com/algebra/linear_equation/slope-of-a-line.php
https://www.youtube.com/watch?v=Z31F_75C_VE

SAT Connection

Heart of Algebra

1. Create, solve, or interpret a linear expression or equation in one variable that represents a context.

Example:

$$h = 3a + 28.6$$

$m = \frac{\text{change in height}}{\text{change in age}} \Rightarrow \frac{3}{1}$

A pediatrician uses the model above to estimate the height h of a boy, in inches, in terms of the boy's age a, in years, between the ages of 2 and 5. Based on the model, what is the estimated increase, in inches, of a boy's height each year?

for every 1 year, height changes 3 inches.

A) 3
B) 5.7
C) 9.5
D) 14.3

Solution

Polynomial Functions

A polynomial function of degree n (where n is a nonnegative integer) is written as:

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$$f(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x^1 + a_0$$

Example:
Write a sample polynomial:

$f(x) = 23x^7 + 20x^6 + 17x^5 + 14x^4 - 10x^3 + 2$

Degree:

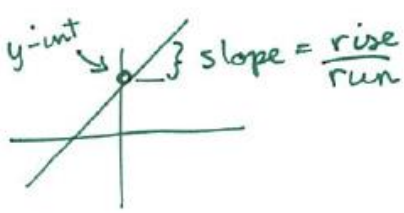
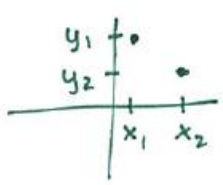
Leading coefficient:

Name	Form	Degree
Zero Function	$f(x) = 0$	no degree
Constant Function	$f(x) = c$ ← constant	0
Linear Function	$f(x) = ax + b$	1
Quadratic Function	$f(x) = ax^2 + bx + c$	2

Linear Function

$f(x) = ax + b$

One of the forms below is needed to write a linear function.

Slope-Intercept Form	Point-Slope Form
$y = mx + b$ slope, y-intercept  	$y - y_1 = m(x - x_1)$ slope point  2 pts (x_1, y_1) (x_2, y_2) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example: Write an equation for the linear function, $f(x)$, where $f(1) = 3$ and $f(-2) = 9$.

pt $(1, 3)$ x_1 y_1 pt $(-2, 9)$ x_2 y_2

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{9-3}{-2-1} = \frac{6}{-3} = -2$

$y - y_1 = m(x - x_1)$

* write equation →
* sub in m, x, y

$y - 3 = -2(x - 1)$

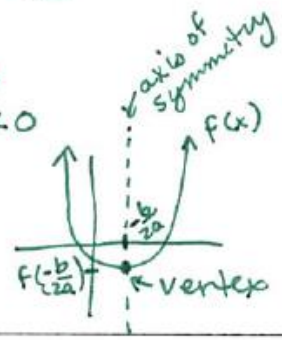
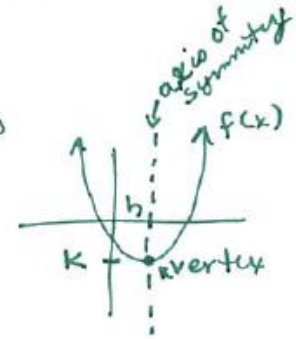
$+3 \qquad +3$

* solve for y and write as function →

$y = -2(x-1) + 3$

$\boxed{f(x) = -2(x-1) + 3}$ or $\boxed{f(x) = -2x + 5}$

Quadratic Function

Standard Form	Vertex Form
$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$
vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	vertex: (h, k)
axis of symmetry: $x = -\frac{b}{2a}$	axis of symmetry: $x = h$
opens up → $a > 0$ down → $a < 0$	opens up → $a > 0$ down → $a < 0$
	

Find the vertex, find the axis of symmetry, and describe the opening of the function:

Example 1:

$f(x) = 3(x+2)^2 - 7$

vertex: $(-2, -7)$

axis of symmetry: $x = -2$

opens up b/c $a > 0$

$a = 3$ ... ☺

Example 2:

$g(x) = -2x^2 + 7x - 3$

$a \quad b \quad c$

vertex: $\left(\frac{-7}{2(-2)}, g\left(\frac{-7}{2(-2)}\right)\right)$

$(1.75, g(1.75))$

$(1.75, 3.125)$

$g(1.75) = -2(1.75)^2 + 7(1.75) - 3 = 3.125$

axis of symmetry: $x = 1.75$

opens down b/c $a < 0$

$a = -2$ ... ☺

Example 3:

$h(x) = 8 + 2x - x^2$ → $h(x) = -x^2 + 2x + 8$

vertex: $\left(\frac{-2}{2(-1)}, h\left(\frac{-2}{2(-1)}\right)\right)$

$(1, h(1))$

$(1, 9)$

$h(1) = -(1)^2 + 2(1) + 8 = 9$

axis of symmetry: $x = 1$

opens down b/c $a < 0$

$a = -1$ ... ☺

Example 4:
Write the quadratic equation with the vertex $(2, -7)$ and the point $(4,5)$.

h k x y

$f(x) = a(x-h)^2 + k$

$5 = a(4-2)^2 + -7$ ← sub in vertex, point and solve for a

$5 = a(2)^2 - 7$

$5 = 4a - 7$

$+7 \quad +7$

$12/4 = 4a/4$

$3 = a$

$\boxed{f(x) = 3(x-2) - 7}$ ← write function w/ values of a, h, k

More Practice

Writing Linear Equations

http://www.mathsisfun.com/algebra/linear-equations.html
http://www.mathplanet.com/education/algebra-1/formulating-linear-equations/writing-linear-equations-using-the-slope-intercept-form
https://www.khanacademy.org/math/algebra/two-var-linear-equations/point-slope/v/idea-behind-point-slope-form
https://www.youtube.com/watch?v=eHPTyYbNmx4

Quadratic Functions

http://mathbitsnotebook.com/Algebra1/Quadratics/QDVertexForm.html
http://www.purplemath.com/modules/grphquad2.htm
http://jwilson.coe.uga.edu/emt668/emat6680.f99/jones/instructional%20unit/writingquads.html
https://www.youtube.com/watch?v=0vSVCN3kJTY
https://www.youtube.com/watch?v=Pk-vBgl67JI
https://www.youtube.com/watch?v=BYlWhtgUwJI

Homework Assignment

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SAT Connection

Solution

Choice A is correct. In the equation $h = 3a + 28.6$, if a, the age of the boy, increases by 1, then h becomes $h = 3(a + 1) + 28.6 = 3a + 3 + 28.6 = (3a + 28.6) + 3$. Therefore, the model estimates that the boy's height increases by 3 inches each year.

Alternatively: The height, h, is a linear function of the age, a, of the boy. The coefficient 3 can be interpreted as the rate of change of the function; in this case, the rate of change can be described as a change of 3 inches in height for every additional year in age.

Choices B, C, and D are incorrect and are likely to result from common errors in calculating the value of h or in calculating the difference between the values of h for different values of a.