2013

Answer Key for AP Calculus AB Practice Exam, Section I

Question 1: D

Question 2: B

Question 3: B

Question 4: E

Question 5: D

Question 6: B

Question 7: C

Question 8: C

Question 9: D

Question 10: A

Question 11: D

Question 12: E

Question 13: E

Question 14: A

Question 15: C

Question 16: D

Question 17: B

Question 18: B

Question 19: B

Question 20: D

Question 21: A

Question 22: E

Question 23: B

Question 24: A

Question 25: C

Question 26: E

Question 27: D

Question 28: B

2013 | AB | Non Calculator

1) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-4}$

$= \lim_{x\to 2} \frac{(x+3)(x-2)}{(x+2)(x-2)}$

$= \lim_{x\to 2} \frac{x+3}{x+2}$

$= \frac{5}{4}$ $\boxed{D}$

2) $f(x) = x^3 - x^2 + x - 1$

$f'(x) = 3x^2 - 2x + 1$

$f'(2) = 3(2)^2 - 2(2) + 1$

$= 12 - 4 + 1$

$= 9$

$\boxed{B}$

3) $\int_0^4 x e^{x^2}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$u(0) = 0$

$u(4) = 16$

$\int_0^{16} x \cdot e^u \cdot \frac{du}{2x}$

$= \frac{1}{2}\int_0^{16} e^u\,du$

$\boxed{B}$

4) $y - y_1 = m(x - x_1)$

$2x - 3(y \cdot 1 + x \cdot \frac{dy}{dx}) = 0$

$2(1) - 3(-3 + \frac{dy}{dx}) = 0$

$2 + 9 - 3\frac{dy}{dx} = 0$

$-3\frac{dy}{dx} = -11$

$\frac{dy}{dx} = +\frac{11}{3}$

$y + 3 = +\frac{11}{3}(x - 1)$ $\boxed{E}$

5) g dec → $g' < 0$

$g'(x) = x^2 + 3x - 70$

$0 = (x+10)(x-7)$

$x = -10,\ x = 7$

g': $+$ $-$ $+$; -11, -10, 0, 7, 8

g dec on $(-10, 7)$ $\boxed{D}$

6) $\int_2^4 \frac{dx}{5-3x}$

$u = 5 - 3x$

$\frac{du}{dx} = -3$

$\frac{du}{-3} = dx$

$u(2) = -1$

$u(4) = -7$

$\int_{-1}^{-7} \frac{1}{u} \cdot \frac{du}{-3}$

$-\frac{1}{3} \ln|u| \Big|_{-1}^{-7}$

$-\frac{1}{3}(\ln|-7| - \ln|-1|)$

$-\frac{1}{3}\ln 7$ $\boxed{B}$

7) inst rate of change → $f'(x)$

$f'(x) = 3x^2 - 12x + 8$

$f'(3) = 3(3)^2 - 12(3) + 8$

$= 27 - 36 + 8$

$= 35 - 36$

$= -1$

$\boxed{C}$

8) speed dec → $v(t)$ and $a(t)$ have diff signs

$v(t)$: $+$ $-$; 0, k, m

$v'(t) = a(t)$: $+$ $-$ $+$; 0, j, l

$v(t) > 0$ & $a(t) < 0$ on (j, k)

$v(t) < 0$ & $a(t) > 0$ on (l, m) $\boxed{C}$

So, speed dec on $(j,k) \cup (l,m)$

9) $f(x) = \frac{(x-2)^2(x+3)}{(x-2)(x+1)}$

hole @ $x = 2$

asymptote @ $x = -1$

$\boxed{D}$

10) $x(3) = -2 + \int_0^3 (3t^2 - 4)\,dt$

$= -2 + (t^3 - 4t)\Big|_0^3$

$= -2 + 3^3 - 4(3) - 0$

$= -2 + 27 - 12$

$= 27 - 14$

$= 13$

$\boxed{A}$

11) f(x) conc. down → $f'' < 0$

$f'(x) = \frac{d}{dx}\int_0^x (2t^3 - 15t^2 + 36t)\,dt$

$f'(x) = 2x^3 - 15x^2 + 36x$

$f''(x) = 6x^2 - 30x + 36$

$0 = 6(x^2 - 5x + 6)$

$0 = 6(x-2)(x-3)$

$x = 2,\ x = 3$

f'': $+$ $-$ $+$; 0, 2, 2.5, 3, 5

f conc. down on $(2, 3)$ $\boxed{D}$

12) I. $\lim_{x\to\infty} \frac{\ln x}{x^{99}} = \frac{\ln\infty}{\infty^{99}}$ → grows faster; so, $\frac{\#}{\infty} = 0$

II. $\lim_{x\to\infty} \frac{e^x}{\ln x} = \frac{e^\infty}{\ln\infty}$ → grows faster; So, $\frac{\infty}{\#} = \infty$

III. $\lim_{x\to\infty} \frac{x^{99}}{e^x} = \frac{\infty^{99}}{e^\infty}$ → grows faster; so, $\#/\infty = 0$

I and III. $\boxed{E}$

13) $f(0) = -5$

$f'(x) \le 3$

$f'(2) = \frac{f(2) - f(0)}{2 - 0}$

$= \frac{f(2) - -5}{2} \le 3$

$= \frac{f(2) + 5}{2} \le 3$

$f(2) + 5 \le 6$

$f(2) \le 1$

So, 2 is not possible $\boxed{E}$

14) f diff'able, so f also cont.

$\lim_{x\to 1^-}(x + b) = \lim_{x\to 1^+}(ax^2)$

$1 + b = a(1)^2$

$1 + b = a$

$1 + b = \frac{1}{2}$

$b = -\frac{1}{2}$

$f'(x) = \begin{cases} 1 & x \le 1 \\ 2ax & x > 1 \end{cases}$

$\lim_{x\to 1^-} 1 = \lim_{x\to 1^+} 2ax$

$1 = 2a$

$\frac{1}{2} = a$

$\boxed{A}$

15) $k'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$k'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2}$

$= \frac{2(5) - (-1)(-2)}{(2)^2}$

$= \frac{10 - 2}{4} = \frac{8}{4} = 2$ $\boxed{C}$

16) $y = 5x(x^2+1)^{1/2}$

$\frac{dy}{dx} = (x^2+1)^{1/2}(5) + 5x\left[\frac{1}{2}(x^2+1)^{-1/2}(2x)\right]$

$\frac{dy}{dx}\Big|_{x=3} = (10)^{1/2} \cdot 5 + 15\left[(10)^{-1/2}(3)\right]$

$= 5\sqrt{10} + \frac{45}{\sqrt{10}}$ $\boxed{D}$

17) $\lim_{h\to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$

def. of derivative where $f(x) = \arcsin x$

$f'(x) = \frac{1}{\sqrt{1-x^2}} = 2$

$1 = 2\sqrt{1-x^2}$

$\frac{1}{2} = \sqrt{1-x^2}$

$\frac{1}{4} = 1 - x^2$

$-3/4 = -x^2$

$3/4 = x^2$

$\pm\frac{\sqrt{3}}{2} = x$

$\boxed{B}$

18) $\ln(2x + y) = x + 1$

$\frac{1}{2x+y} \cdot (2 + \frac{dy}{dx}) = 1$

$2 + \frac{dy}{dx} = 2x + y$

$\boxed{B}$ $\frac{dy}{dx} = 2x + y - 2$

19) $h(x) = e^x g(x)$

$h'(x) = g(x)e^x + e^x g'(x)$

$h'(-1) = g(-1)e^{-1} + e^{-1}g'(-1)$

$= 3 \cdot \frac{1}{e} + \frac{1}{e} \cdot -6$

$= \frac{3}{e} - \frac{6}{e}$

$= -\frac{3}{e}$ $\boxed{B}$

$g'(-1) = \frac{g(0) - g(-1)}{0 - -1}$

$= \frac{-3 - 3}{1}$

$= -6$

20. $\frac{d}{dx}\int_0^{2x} \ln(t^3+1)\,dt$

$= \ln((2x)^3+1)\cdot 2$

$\boxed{D}$ $= 2\ln(8x^3+1)$

21. $\int_0^7 f(x)\,dx$

$= \frac{1}{2}(1+2)(2) + 2(2) + \frac{1}{2}(1)(2) + -\frac{1}{2}(2)(2)$

$= 3+4+1-2$

$= 6$ $\boxed{A}$

22. $\frac{f(9)-f(6)}{9-6} = -\frac{3}{2}$

but $f'(c) \neq -\frac{3}{2}$, $\therefore$ MVT doesn't apply... So f not diff'able

$\frac{1}{3}(f(9)-f(6))$

$= \frac{1}{3}\int_6^9 f'(x)\,dx$

$\boxed{E}$

23. $f(0)=1$ $g(1)=0$

$f'(0)=3$ $g'(1)=\frac{1}{3}$

$2x+e^x=1$

if $x=0$

$f'(x) = 2+e^x$

$f'(0) = 2+e^0$

$= 3$

$\boxed{B}$

24. abs min value → lowest y-value.

$g'(x) = 12x^2+6x-6$

$0 = 6(2x^2+x-1)$

$0 = 6(2x-1)(x+1)$

$x = 1/2,\ x=-1$

g ↗ ↘ ↗

g' + − +

−2 −1 0 1/2 1

abs min @ $x = 1/2$ or endpt $x=-2$

$g(1/2) = 4(1/2)^3+3(1/2)^2-6(1/2)+1$

$= 4(1/8)+3(1/4)-3+1$

$= 1/2 + 3/4 - 3+1$

$= 2/4+3/4-2$

$= 5/4 - 8/4$

$= -3/4$

$g(-2) = 4(-2)^3+3(-2)^2-6(-2)+1$

$= 4(-8)+3(4)+12+1$

$= -32+12+12+1$

$= -32+25$

$= -7$

abs max value −7

$\boxed{A}$

25. $\frac{dy}{dx} = e^y\cdot e^x$

$dy = e^y\cdot e^x\,dx$

$\int e^{-y}\,dy = \int e^x\,dx$

$u=-y$

$\frac{du}{dy} = -1$

$\frac{du}{-1} = dy$

$-e^{-y} = e^x + C$

$-e^{-(-\ln 4)} = e^0 + c$

$-e^{\ln 4} = 1+c$

$-4 = 1+c$

$-5 = c$

$-e^{-y} = e^x + -5$

$e^{-y} = -e^x+5$

$-y = \ln(-e^x+5)$

$y = -\ln(-e^x+5)$

$\boxed{C}$

26. If (A) $h(x) = \frac{2}{3}(1+x^3)^{3/2}$

$h'(x) = (1+x^3)^{1/2}\cdot 3x^2$ ×

(B) $h(x) = \dfrac{\frac{2}{3}(1+x^3)^{3/2}}{3x^2}$

$h'(x) = \dfrac{3x^2(1+x^3)^{1/2}\cdot 3x^2 - \frac{2}{3}(1+x^3)^{3/2}\cdot 6x}{(3x^2)^2}$ ×

(C) $\frac{d}{dx}\int_0^{1+x^3}\sqrt{t}\,dt$

$= \sqrt{1+x^3}\cdot 3x^2$ ×

(D) $\frac{d}{dx}\int_0^{x^3}\sqrt{1+t}\,dt$

$= \sqrt{1+x^3}\cdot 3x^2$ ×

(E) $\frac{d}{dx}\int_0^x \sqrt{1+t^3}\,dt$

$\sqrt{1+x^3}$ ✓ $\boxed{E}$

27. avg Value $= \frac{1}{2-0}\int_0^2 16+7\cos(\frac{\pi t}{4})\,dt$

$u = \frac{\pi}{4}t$

$\frac{du}{dt} = \frac{\pi}{4}$

$\frac{4}{\pi}du = dt$

$u(2) = \pi/2$

$u(0) = 0$

$= \frac{1}{2}\int_0^{\pi/2}(16+7\cos u)(\frac{4}{\pi}dt)$

$= \frac{2}{\pi}\int_0^{\pi/2}(16+7\cos u)\,du$

$= \frac{2}{\pi}(16u+7\sin u)\Big|_0^{\pi/2}$

$= \frac{2}{\pi}\left[16\cdot\frac{\pi}{2}+7\sin\frac{\pi}{2} - (16\cdot 0+7\sin 0)\right]$

$= \frac{2}{\pi}(8\pi+7)$

$\boxed{D}$ $= 16+\frac{14}{\pi}$

28. pt inf → f'' changes signs

f conc up to f conc. down → f'' changes pos to neg.

$f(x) = \sin x+\cos x$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

$0 = -\sin x - \cos x$

$\sin x = -\cos x$

$x = 3\pi/4,\ 7\pi/4$

f'' − + −

0 $\frac{3\pi}{4}$ π $\frac{7\pi}{4}$ 2π

$\boxed{B}$

@ $x = 3\pi/4$, f changes from conc. down to conc. up