2014

Answer Key for AP Calculus AB Practice Exam, Section I

Question 1: A

Question 2: B

Question 3: D

Question 4: E

Question 5: C

Question 6: E

Question 7: A

Question 8: D

Question 9: A

Question 10: E

Question 11: D

Question 12: C

Question 13: C

Question 14: D

Question 15: A

Question 16: B

Question 17: E

Question 18: B

Question 19: B

Question 20: B

Question 21: C

Question 22: D

Question 23: E

Question 24: B

Question 25: D

Question 26: A

Question 27: D

Question 28: A

2014 AB NonCalculator

(1) $\int_2^x (3t^2-1)\,dt$

$= (t^3 - t)\Big|_2^x$

$= x^3 - x - (2^3 - 2)$

$= x^3 - x - 6$ $\boxed{A}$

(2) $y = \ln(2x)$

$y' = \frac{1}{2x}\cdot 2$

$y' = \frac{1}{x}$

$y'(4) = \frac{1}{4}$ $\boxed{B}$

(3) $f(x) = 4x^{-2} + \frac{1}{4}x^2 + 4$

$f'(x) = -8x^{-3} + \frac{1}{2}x$

$f'(2) = -8(2)^{-3} + \frac{1}{2}(2)$

$= -8(\frac{1}{8}) +$

$= -1 + 1$

$= 0$ $\boxed{D}$

(4) $\int_1^2 \frac{dx}{2x+1}$

$u = 2x+1$

$\frac{du}{dx} = 2$

$\frac{1}{2}du = dx$

$u(2) = 5$

$u(1) = 3$

$\int_3^5 \frac{1}{u}\cdot\frac{1}{2}du$

$= \frac{1}{2}\int_3^5 \frac{1}{u}du$

$= \frac{1}{2}\ln|u|\Big|_3^5$

$= \frac{1}{2}(\ln 5 - \ln 3)$ $\boxed{E}$

(5)

I. $\lim_{x\to 2^-} f(x) = -1$, $f(2) = -1$ $\} = ✓$

II. $\lim_{x\to 6^-} f(x) = -1$, $\lim_{x\to 6^+} f(x) = -1$ $\} = ✓$

III. $\lim_{x\to 6} f(x) = -1$, $f(6) = -3$ $\} \neq$ X

$\boxed{C}$ I and II only

(6) $\frac{d}{dt}(\sin^3(x^2))$ (double chain ... ☺)

$= \frac{d}{dx}((\sin x^2)^3)$

$= 3(\sin x^2)^2 \cdot \cos x^2 \cdot 2x$

$= 6x \sin^2 x^2 \cos x^2$ $\boxed{E}$

(7) $\lim_{x\to\infty} \frac{x^3}{e^{3x}} = \frac{\infty}{e^\infty} = \frac{\infty}{\infty}$?

e^{3x} grows bigger than x^3, so num < den.

$\lim_{x\to\infty}\frac{x^3}{e^{3x}} = 0$ $\boxed{A}$

(8) $\int_{\pi/6}^{\pi/2} \sin^5(2x)\cos(2x)\,dx$

$u = \sin(2x)$

$\frac{du}{dx} = \cos(2x)\cdot 2$

$\frac{du}{2\cos(2x)} = dx$

$u(\pi/2) = \sin(2\cdot\frac{\pi}{2}) = \sin\pi = 0$

$u(\pi/6) = \sin(2\cdot\pi/6) = \sin\pi/3 = \sqrt{3}/2$

$\int_{\sqrt{3}/2}^{0} u^5 \cdot \cos 2x \cdot \frac{du}{2\cos 2x}$

$= \frac{1}{2}\int_{\sqrt{3}/2}^{0} u^5\,du$ $\boxed{D}$

(9) rel. max → f' changes pos to neg

$f' = 0$ @ $x = 0,\ x = 3,\ x = -1$

f': + (−2) | − (−1/2) | + (1) | + (4), at -1, 0, 3

↓ rel max @ $x = -1$

$\boxed{A}$

(10) cont when $\lim_{x\to 2} f(x) = f(2)$

$\lim_{x\to 2}\left(\frac{x^2-7x+10}{b(x-2)}\right) = b$

$\lim_{x\to 2}\frac{(x-5)(x-2)}{b(x-2)} = b$

$\lim_{x\to 2}\frac{x-5}{b} = b$

$\frac{-3}{b} = b$

$-3 = b^2$

$\sqrt{-3} = b$ ← no imaginary #s

so $\boxed{E}$

(11) max value → abs max

check rel. max & endpts.

f': + (0 to 3) | − (3 to 5.5) | +, at 0, 3, 5.5

→ rel. max @ $x = 3$

$f(0) = 1$

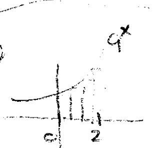
$f(3) = 1 + \int_0^3 f'(t)\,dt$ (area)

$= 1 + \frac{1}{2}(3)(2)$

$= 1 + 3$

$f(3) = 4$

abs max value is 4 $\boxed{D}$

(12) 

$\Delta x = \frac{2-0}{4}$

$\Delta x = \frac{1}{2}$

x	0	1/2	1	3/2	2
f(x)	1	3	9	27	81

$\int_0^2 f(x)\,dx = \frac{1}{2}(81 + 27 + 9 + 3)$

$= \frac{1}{2}(90 + 30)$

$= \frac{1}{2}(120)$

$\boxed{C}$ $= 60$

(13) $\frac{d}{dx}\left(\frac{x+1}{x^2+1}\right)$ quotient rule, lo di-hi, lo²-hi do

$= \frac{(x^2+1)(1) - (x+1)(2x)}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2-2x}{(x^2+1)^2}$

$= \frac{-x^2-2x+1}{(x^2+1)^2}$

$\boxed{C}$

(14) $x(\frac{\pi}{2}) = x(0) + \int_0^{\pi/2} v(t)\,dt$

$= 4 + \int_0^{\pi/2}\sin(2t)\,dt$

$u = 2t$, $\frac{du}{dt} = 2$, $\frac{1}{2}du = dt$

$u(\pi/2) = 2(\pi/2) = \pi$

$u(0) = 0$

$= 4 + \frac{1}{2}\int_0^{\pi}\sin u\,du$

$= 4 - (\frac{1}{2}\cos u)\Big|_0^{\pi}$

$= 4 - \frac{1}{2}(\cos\pi - \cos 0)$

$= 4 - \frac{1}{2}(-1-1)$

$= 4 - \frac{1}{2}(-2)$

$= 4 + 1$

$= 5$ $\boxed{D}$

(15) inc → $y' > 0$ (chain rule ... ☺)

$y = g(x^3 - 6x^2)$

$y' = g'(x^3-6x^2)\cdot(3x^2-12x)$

$0 = g'(x^3-6x^2)\cdot(3x^2-12x)$

$\underbrace{g'(x^3-6x^2) = 0}_{\text{always inc,}}$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0,\ x = 4$

y': + (−1) | − (1) | + (5), at 0, 4

$y' > 0$ on $(-\infty, 0)\cup(4, \infty)$

$\boxed{A}$

(16) $\lim_{x \to 3^-} \frac{|x-3|}{x-3}$

from left, so $|x-3|$ bc $x-3$ neg.

$\lim_{x \to 3^-} \frac{-(x-3)}{x-3}$

$= \lim_{x \to 3^-} -1$

$= -1$ $\boxed{B}$

(17) $f(x) = ae^{-ax}$

$f'(x) = ae^{-ax} \cdot -a$

$= -a^2 e^{-ax}$

$\boxed{E}$

(18) $\frac{dy}{dx} = xy$

$\int \frac{1}{y}\,dy = \int x\,dx$ ✓

$\ln|y| = \frac{1}{2}x^2 + C$ ✓

$|y| = e^{\frac{1}{2}x^2 + C}$ ✗

student didn't put +C up in exponent

$\boxed{B}$

(19) pt of inf → y'' change signs

$y = 3x^5 + 10x^4$

$y' = 15x^4 + 40x^3$

$y'' = 60x^3 + 120x^2$

$0 = 60x^2(x+2)$

$x = 0,\ x = -2$

y'': $-$ | $+$ | $+$ at -3, -2, -1, 0, 1

pt inf @ $x = -2$ $\boxed{B}$

(20) $\lim_{x \to 2} \frac{\ln(x+3) - \ln(5)}{x-2}$

alternate form of derivative

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$f'(2) = \lim_{x \to 2} \frac{\ln(x+3) - \ln 5}{x-2}$

$f(x) = \ln(x+3)$

$f'(x) = \frac{1}{x+3}$

$f'(2) = \frac{1}{5}$

$\boxed{B}$

(21) Related Rates

$\frac{dx}{dt} = -1$ unit/min

$\frac{dy}{dt} = 4$ units/min

$x = 6$

$y = 20$

$\frac{du}{dt} = ?$

$u = x^2 y$

$\frac{du}{dt} = y\left(2x\frac{dx}{dt}\right) + x^2\frac{dy}{dt}$

$\frac{du}{dt} = 20(2(6)(-1)) + 6^2(4)$

$= -240 + 144$

$= -96$ $\boxed{C}$

(22) f dec → $f' < 0$

f conc up → $f'' > 0$

$f(x) = 2x^3 - 3x^2 - 12x + 15$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2,\ x = -1$

f': $+$ | $-$ | $+$ at -2, -1, 0, 2, 3

dec on $(-1, 2)$

$f''(x) = 12x - 6$

$0 = 12x - 6$

$\frac{1}{2} = x$

f'': $-$ | $+$ at 0, $\frac{1}{2}$, 1

conc. up on $(\frac{1}{2}, \infty)$

dec and conc. up on $(\frac{1}{2}, 2)$ $\boxed{D}$

(23) $f(x) = \begin{cases} 3x+5 & x < -1 \\ -x^2+3 & x \geq -1 \end{cases}$

$f'(x) = \begin{cases} 3 & x < -1 \\ -2x & x \geq -1 \end{cases}$

$\lim_{x \to -1^-} f'(x) = 3$

$\lim_{x \to -1^+} f'(x) = 2$

$\neq$ so $f'(-1)$ DNE

$\boxed{E}$

(24) H. A. → $\lim_{x \to \infty} f(x)$

$\lim_{x \to \infty} \frac{(3x+8)(5-4x)}{(2x+1)^2}$

$\lim_{x \to \infty} \frac{-12x^2 + \cdots}{4x^2 + \cdots}$

deg N = deg D so look at coefficients

$\frac{-12}{4} = -3$

$y = -3$ $\boxed{B}$

(25) $y = x^2 - 2x$

$\frac{dy}{du} = 2x\frac{dx}{du} - 2\frac{dx}{du}$

$= 2x(\frac{1}{2}) - 2(\frac{1}{2})$

$= x - 1$ $\boxed{D}$

$u = 2x + 1$

$u - 1 = 2x$

$\frac{1}{2}u - \frac{1}{2} = x$

$\frac{1}{2} = \frac{dx}{du}$

(26) $\frac{d}{dx} \int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt$

2nd FTC

$= \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2}$

$= \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}(1+x)}$ $\boxed{A}$

(27) particle rest → velocity = 0

$x(t) = \sin(2\pi t) - 2\pi t$

$v(t) = x'(t) = \cos(2\pi t) \cdot 2\pi + 2\pi$

$0 = 2\pi\cos(2\pi t) + 2\pi$

$0 = 2\pi[\cos(2\pi t) + 1]$

$\cos(2\pi t) + 1 = 0$

$\cos(2\pi t) = -1$

$2\pi t = \pi, 3\pi, 5\pi, \ldots$

$t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

not in interval $[0, 1]$

$\boxed{D}$

(28) $\frac{dy}{dx} = 0$ @ $x = 0$, so A, B, or D

$\frac{dy}{dx} = 0$ @ $y = 1$, so A or D

when $y > 1$ and $x < 0$ (in Quad II),

$\frac{dy}{dx} < 0$, so only A

$\boxed{A}$