2015

Answer Key for AP Calculus AB Practice Exam, Section I

Question 1: B

Question 2: A

Question 3: E

Question 4: A

Question 5: B

Question 6: B

Question 7: D

Question 8: C

Question 9: B

Question 10: A

Question 11: C

Question 12: A

Question 13: E

Question 14: D

Question 15: C

Question 16: C

Question 17: C

Question 18: B

Question 19: C

Question 20: C

Question 21: B

Question 22: A

Question 23: B

Question 24: D

Question 25: B

Question 26: C

Question 27: B

Question 28: A

2015 AB

Non Calculator

① $\int (5e^{2x} + \frac{1}{x})\,dx$

$5\int e^{2x}\,dx + \int \frac{1}{x}\,dx$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$5\int e^{u} \cdot \frac{1}{2}du$

$\frac{5}{2}e^{u} + \ln|x| + C$

$\boxed{B}$ $\frac{5}{2}e^{2x} + \ln|x| + C$

② $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$

$f(x) = x^{1/2} + 3x^{-1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}$

$f'(4) = \frac{1}{2}(4)^{-1/2} - \frac{3}{2}(4)^{-3/2}$

$= \frac{1}{2\sqrt{4}} - \frac{3}{2}\cdot\frac{1}{\sqrt{4^3}}$

$= \frac{1}{4} - \frac{3}{2}\cdot\frac{1}{8}$

$= \frac{1}{4} - \frac{3}{16}$

$\boxed{A}$ $= \frac{4}{16} - \frac{3}{16} = \frac{1}{16}$

③ $\int x^2(x^3+5)^6\,dx$

$u = x^3 + 5$
$du = 3x^2\,dx$
$\frac{du}{3x^2} = dx$

$\int x^2 \cdot u^6 \cdot \frac{du}{3x^2}$

$\frac{1}{3}\int u^6\,du$

$\frac{1}{3}\cdot\frac{1}{7}u^7 + C$

$\boxed{E}$ $\frac{1}{21}(x^3+5)^7 + C$

④ $\int_0^{50} f(x)\,dx = 25(4) + 5(6) + 20(8)$

$= 100 + 30 + 160$

$= 100 + 190$

$= 290$

$\boxed{A}$

⑤ f cont → need limits =

$\lim_{x\to 2^-} f(x) = 2^2\sin(2\pi)$
$= 4(0) = 0$

$\lim_{x\to 2^+} f(x) = 2^2 + c(2) - 18$
$= 4 + 2c - 18$
$= 2c - 14$

$2c - 14 = 0$
$2c = 14$
$c = 7$

$\boxed{B}$

⑥ $\int(3\sec^2 x + 2)\,dx$

$= 3\int \sec^2 x\,dx + \int 2\,dx$

$= 3\tan x + 2x + C$

$\boxed{B}$

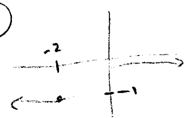
⑦ $\frac{d}{dx}(f(g(x)))$

$= f'(g(x))\cdot g'(x)$

$= 2g(x)\cdot g'(x)$

$f(x) = x^2 - 4$
$f'(x) = 2x$
$f'(g(x)) = 2g(x)$

$\boxed{D}$

⑧ 

g(x) solution to diff'eq @ $(-2,-1)$

$\lim_{x\to\infty} g(x) = 0$

$\boxed{C}$

⑨ $f''(x) = x(x+2)^2$

$0 = x(x+2)^2$

$x = 0,\ x = -2$

f'': $-$ | $-$ | $+$; -3, -2, -1, 0, 1

$\boxed{B}$ f conc up on $(0,\infty)$

⑩ $y = \sin x\cos x$

$\frac{dy}{dx} = \cos x(\cos x) + \sin x(-\sin x)$

$\frac{dy}{dx}\Big|_{x=\pi/3} = (\cos\frac{\pi}{3})^2 - (\sin\frac{\pi}{3})^2$

$= (\frac{1}{2})^2 - (\frac{\sqrt{3}}{2})^2$

$= \frac{1}{4} - \frac{3}{4}$

$\boxed{A}$ $= -\frac{2}{4} = -\frac{1}{2}$

⑪ $\lim_{x\to -3}\frac{x^2-9}{x^2-2x-15}$

$\lim_{x\to -3}\frac{(x-3)(x+3)}{(x-5)(x+3)}$

$\lim_{x\to -3}\frac{x-3}{x-5}$

$= \frac{-6}{-8} = \frac{3}{4}$ $\boxed{C}$

⑫ avg rate change $= \frac{f(b)-f(a)}{b-a}$

$= \frac{f(\pi/2) - f(0)}{\pi/2 - 0}$

$= \frac{\cos(2\cdot\pi/2) - \cos(2\cdot 0)}{\pi/2}$

$= \frac{\cos\pi - \cos 0}{\pi/2}$

$\boxed{A}$ $= \frac{-1-1}{\pi/2} = \frac{-2}{\pi/2} = -\frac{4}{\pi}$

⑬ $y^3 + y = x^2$

$3y^2\frac{dy}{dx} + \frac{dy}{dx} = 2x$

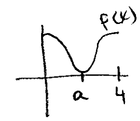
$\frac{dy}{dx}(3y^2+1) = 2x$

$\frac{dy}{dx} = \frac{2x}{3y^2+1}$

$\boxed{E}$

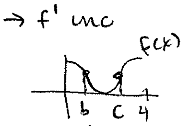
⑭ f dec → f' < 0 on $(0,a)$

f inc → f' > 0 on $(a,4)$



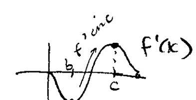
So, C or D

f conc up → f' inc on (b,c)



f conc. down → f' dec on $(0,b)\cup(c,4)$

So, $\boxed{D}$



⑮ I. $\lim_{x\to 1^-} f(x) = 3(1) - 2$
$= 1$

$\lim_{x\to 1^+} f(x) = \ln(3(1)-2)$
$= \ln 1$
$= 0$

$\neq$

II. $f'(x) = \begin{cases} 3 & x<1 \\ \frac{1}{3x-2}\cdot 3 & x\geq 1\end{cases}$

$\lim_{x\to 1^-} f'(x) = 3$

$\lim_{x\to 1^+} f'(x) = \frac{1}{3(1)-2}\cdot 3$
$= 3$

$= \checkmark$

III. f is ~~not~~ diff'able @ $x = 1$ b/c $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ "jump"

II only

$\boxed{C}$

⑯ MVT → $f'(c) = \frac{f(b)-f(a)}{b-a}$

$f'(c) = \frac{f(3)-f(1)}{3-1}$

$f'(c) = \frac{2(3)^3 - 4(3)^2 + 1 - [2(1)^3 - 4(1)^2 + 1]}{2}$

$= \frac{54 - 36 + 1 - (2 - 4 + 1)}{2}$

$= \frac{54 - 36 + 1 - 2 + 4 - 1}{2}$

$= \frac{58 - 38}{2}$

$\boxed{C}$ $= \frac{20}{2} = 10$

(17)

$h = 2$ meters

$v(h) = 3\sqrt{h}$

$v'(h) = 3(\frac{1}{2}h^{-1/2})$

$v'(2) = \frac{3}{2}(2)^{-1/2}$

$= \frac{3}{2\sqrt{2}}$

$\boxed{C}$

(18)

$y(6) = y(0) + \int_0^6 -10e^{-t/2}\,dt$

$u = -t/2$, $du = -\frac{1}{2}dt$, $-2du = dt$

$u(6) = -6/2 = -3$

$u(0) = -0/2 = 0$

$= 20 + -10\int_0^{-3} -2e^u\,du$

$= 20 + 20\int_0^{-3} e^u\,du$

$= 20 + (20e^u)\Big|_0^{-3}$

$= 20 + 20(e^{-3} - e^0)$

$= 20 + 20e^{-3} - 20$

$= 20e^{-3}$

$\boxed{B}$

(19)

pt inf → f'' changes signs

$f'(x) = x^3 - 4x$

$f''(x) = 3x^2 - 4$

$0 = 3x^2 - 4$

$4 = 3x^2$

$\frac{4}{3} = x^2$

$\pm\sqrt{\frac{4}{3}} = x$

$\pm\frac{2}{\sqrt{3}} = x$

f': + − +

(−10) $-\frac{2}{\sqrt{3}}$ (0) $\frac{2}{\sqrt{3}}$ (10)

$\boxed{C}$ inf pt @ $x = \frac{2}{\sqrt{3}}$

(20)

H.A. → $\lim_{x\to\infty} f(x)$

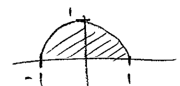
$\lim_{x\to\infty} \frac{(x-4)(2x-3)}{(x-1)^2}$

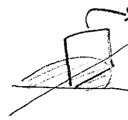
$\lim_{x\to\infty} \frac{2x^2 + \cdots}{x^2 + \cdots}$ deg N = deg D look @ coefficients

$= 2$

$b = 2$ $\boxed{C}$

(21) $y = \sqrt{1-x^2}$





→ Area Square $= (\text{side})^2 = (\sqrt{1-x^2})^2$

$V = \int_{-1}^{1} (\sqrt{1-x^2})^2\,dx$

$= \int_{-1}^{1} (1-x^2)\,dx$

$= 2\int_0^1 (1-x^2)\,dx$ symmetry about y-axis

$= 2(x - \frac{1}{3}x^3)\Big|_0^1$

$= 2(1-\frac{1}{3})$

$= 2(2/3) = 4/3$ $\boxed{B}$

(22)

f dec → $f' < 0$

$f(x) = \frac{kx}{x^2+1}$

quotient rule $\frac{\text{Lodhi-hidlo}}{\text{Lo}^2}$ ☺

$f'(x) = \frac{(x^2+1)(k) - (kx)(2x)}{(x^2+1)^2}$

$= \frac{kx^2 + k - 2kx^2}{(x^2+1)^2}$

$= \frac{k - kx^2}{(x^2+1)^2}$

$= \frac{k(1-x^2)}{(x^2+1)^2}$ } → positive on (−1, 1); } positive for any x

so k needs to be neg

$k > 0$ $\boxed{A}$

(23)

$y' = \frac{d}{dx}(3 - \int_{-1}^{x} e^{-t^3}\,dt)$

$y' = -e^{-x^3}$

$y'(-1) = -e^{-(-1)^3}$

$= -e$

$\boxed{B}$

(24)

$\frac{dy}{dx} = 5y^2$

$\int \frac{1}{y^2}\,dy = \int 5\,dx$

$\int y^{-2}\,dy = \int 5\,dx$ $y(0) = 3$

$-y^{-1} = 5x + C$

$-\frac{1}{3} = 5(0) + C$

$-\frac{1}{3} = C$

$-\frac{1}{y} = 5x - \frac{1}{3}$

$\frac{1}{y} = -5x + \frac{1}{3}$

$\frac{1}{y} = \frac{-15x+1}{3}$

$y = \frac{3}{-15x+1}$ $\boxed{D}$

(25) def. of derivative

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(\frac{\pi}{3}) = \lim_{h\to 0} \frac{\sin(\frac{\pi}{3}+h) - \sin(\frac{\pi}{3})}{h}$

$f(x) = \sin x$ @ $x = \pi/3$

$f'(x) = \cos x$

$f'(\pi/3) = \cos\pi/3$

$= 1/2$ $\boxed{B}$

(26)

1st stops → $v(t) = 0$

$v(t) = 2\cos 3t = 0$

$\cos 3t = 0$

$3t = \pi/2$

$t = \pi/6$

total distance $= \int_0^{\pi/6} |2\cos(3t)|\,dt$

$= \int_0^{\pi/6} 2\cos(3t)\,dt$

$u = 3t$, $du = 3\,dt$, $\frac{1}{3}du = dt$

$u(\pi/6) = \pi/2$, $u(0) = 0$

$= \int_0^{\pi/2} 2\cos u \cdot \frac{1}{3}\,du$

$= \frac{2}{3}\int_0^{\pi/2} \cos u\,du$

$= \frac{2}{3}\sin u\Big|_0^{\pi/2}$

$= \frac{2}{3}(\sin\frac{\pi}{2} - \sin 0)$

$= \frac{2}{3}$

$\boxed{C}$

(27)

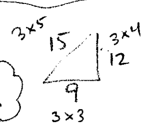
$f(1) = 2$ $f^{-1}(2) = 1$

$f'(1) = -8$ $(f^{-1})'(2) = \frac{1}{-8}$

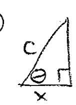
reciprocal

$\boxed{B}$

☺ 3,4,5 △



(28)



$c = 15$ ft

$\frac{dy}{dt} = -2$ ft/sec

$x = 9$ ft

$\frac{d\theta}{dt} = ?$

(use any trig function, but sine is easiest)

$\sin\theta = \frac{y}{c}$ ← constant (ladder size not changing)

$\sin\theta = \frac{1}{c}y$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{c}\frac{dy}{dt}$

$(\frac{9}{15})(\frac{d\theta}{dt}) = \frac{1}{15}(-2)$

$\frac{d\theta}{dt} = \frac{-2}{15}\cdot\frac{15}{9}$

$\frac{d\theta}{dt} = -\frac{2}{9}$

$\boxed{A}$