2016 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) ~~Find the position of the particle at time $t = 0$.~~

(d) ~~Find the total distance the particle travels from time $t = 0$ to time $t = 3$.~~

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a) particle speed up → $a(t)$ + $v(t)$ same signs
particle slow down → $a(t)$ + $v(t)$ diff signs

$v(4) = 2.979$

$a(4) = v'(4) = -1.164$

@ $t=4$, particle is slowing down
b/c $v(4) > 0$ and $a(4) < 0$.

AP ∴ 2 pts: conclusion w/ reason

b) particle change direction
↳ $v(t)$ changes signs.

on $(0,3)$, particle changes direction
@ $t = 2.707$ b/c $v(t) > 0$ on $(0, 2.707)$
and $v(t) > 0$ on $(2.707, 3)$

AP ∴ 1 pt: $t = 2.707$
1 pt: reason

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b) $\frac{dy}{dx} \rightarrow \frac{dy/dt}{dx/dt}$

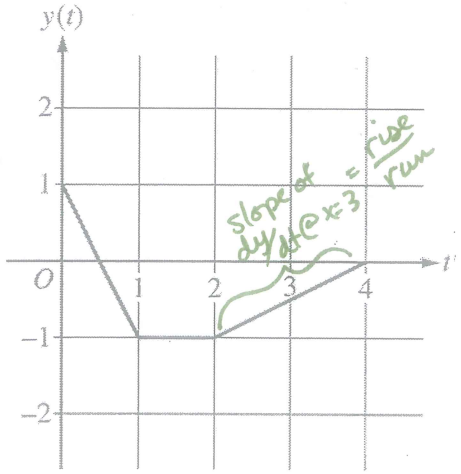
$\left.\frac{dy}{dt}\right|_{t=3} = \frac{1}{2}$

$\left.\frac{dx}{dt}\right|_{t=3} = 9.956$

slope of tangent line $= \frac{1/2}{9.956}$ at $t=3$

$= \boxed{0.050}$

AP 1 pt: slope



c) Speed $= |v(t)|$
$= \sqrt{(dx/dt)^2 + (dy/dt)^2}$

$\text{speed}|_{t=3} = \sqrt{(x'(3))^2 + (y'(3))^2}$

$= \boxed{9.969}$

AP 1 pt: expression for speed
1 pt: answer

2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin\left(3t^2\right)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

~~(a) Find the position of the particle at $t = 3$.~~

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

~~(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.~~

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

$y - y_1 = m(x - x_1)$

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

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a) $y - k(3) = k'(3)(x-3)$

$k(x) = f(g(x))$

$k(3) = f(g(3))$

$= f(6)$

$= 4$

$k'(x) = g'(x) \cdot f'(g(x))$

$k'(3) = g'(3) \cdot f'(g(3))$

$= 2 \cdot f'(6)$

$= 2 \cdot 5$

$= 10$

$\boxed{y - 4 = 10(x-3)}$

2 pts → slope @ $x=3$
1 pt → eq. of tangent line

AP

b) $h(x) = \dfrac{g(x)}{f(x)}$

$h'(x) = \dfrac{f(x)\,g'(x) - g(x)\,f'(x)}{(f(x))^2}$

$h'(1) = \dfrac{f(1)\,g'(1) - g(1)\,f'(1)}{(f(1))^2}$

$= \dfrac{-6(8) - 2(3)}{(-6)^2}$

$= \dfrac{-54}{36}$

$\boxed{h'(1) = -\dfrac{3}{2}}$

2 pts → $h'(1)$ expression
1 pt → answer

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