3.6

Slopes of Parametric Curves
(Parametrized)

- same as slope of $\frac{dy}{dx}$

$$x = f(t) \qquad y = g(t)$$

$$\frac{dx}{dt} = f'(t) \qquad \frac{dy}{dt} = g'(t)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dx}$$

ex: $x = 2\cos t \quad y = 2\sin t$ @ $t = \pi/4$. Find eq of tangent Line

$$x\left(\frac{\pi}{4}\right) = 2\cos\frac{\pi}{4} = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$y\left(\frac{\pi}{4}\right) = 2\sin\frac{\pi}{4} = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - \sqrt{2} = -1\left(x - \sqrt{2}\right)}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-2\sin t}$$

$$\left.\frac{dy}{dx}\right|_{t=\pi/4} = \frac{2\cos\pi/4}{-2\sin\pi/4} = \frac{2\left(\frac{\sqrt{2}}{2}\right)}{-2\left(\frac{\sqrt{2}}{2}\right)} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$$

Slopes of Polar Curves

- same as slope $\rightarrow \frac{dy}{dx}$

- convert from polar to rectangular (Cartesian)

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\frac{dx}{d\theta} = -r\sin\theta \qquad \frac{dy}{d\theta} = r\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{r\cos\theta}{-r\sin\theta}$$

$$\frac{dy}{dx} = -\cot\theta$$

~~Limaçon~~

ex: Find the <u>slope of the curve</u> $\frac{dy}{dx}$ $r = 2\sin 3\theta$ at $\theta = \pi/6$ and write eq. of tangent line.

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$x = \underline{2\sin(3\theta)}\,\underline{\cos\theta} \qquad y = \underline{2\sin 3\theta}\,\underline{\sin\theta}$$

$$\frac{dx}{d\theta} = \cos\theta\left[2\cos(3\theta)\cdot 3\right] + 2\sin 3\theta\,(-\sin\theta)$$

$$= 6\cos\theta\cos 3\theta - 2\sin 3\theta\sin\theta$$

$$\frac{dy}{d\theta} = \sin\theta\left[2\cos(3\theta)\cdot 3\right] + 2\sin 3\theta\cdot\cos\theta$$

$$= 6\sin\theta\cos 3\theta + 2\sin 3\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{6\sin\theta\cos 3\theta + 2\sin 3\theta\cos\theta}{6\cos\theta\cos 3\theta - 2\sin 3\theta\sin\theta}$$

$3\cdot\frac{\pi}{6}$

$\frac{\pi}{2}$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = \frac{6\left(\frac{1}{2}\right)(0) + 2(1)\left(\frac{\sqrt{3}}{2}\right)}{6\left(\frac{\sqrt{3}}{2}\right)(0) - 2(1)\left(\frac{1}{2}\right)}$$

$$= \frac{\sqrt{3}}{-1} = -\sqrt{3}$$ slope of the curve