3.7 Implicit Differentiation

Implicit
(implied)
variable deriving is not solved for

ex: $y^2 - 6y = 5x$

Explicit
(stated)
variable is solved for

ex: $y = 3x + 5$

ex: Find $\frac{dy}{dx}$.

$$y^2 - 6y = 5x$$

$$2y\frac{dy}{dx} - 6\frac{dy}{dx} = 5\frac{dx}{dx}$$

$$2y\frac{dy}{dx} - 6\frac{dy}{dx} = 5$$

$$\frac{dy}{dx}(2y-6) = 5$$

$$\boxed{\frac{dy}{dx} = \frac{5}{2y-6}}$$

Differentiate Implicitly

① derive each term w/ respect to x (each term gets $\frac{d\,\text{variable}}{dx}$)

② get $\frac{dy}{dx}$'s on one side

③ factor out $\frac{dy}{dx}$ and solve for $\frac{dy}{dx}$.

Find $\frac{dy}{dx}$.

ex: $x^2 + y^2 = 36$

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{2y\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

ex: $x^2 y = -2 + y$

product

$$\underbrace{y}_{g} \cdot \underbrace{2x\frac{dx}{dx}}_{f'} + \underbrace{x^2}_{f} \cdot \underbrace{1\frac{dy}{dx}}_{g'} = 0 + 1\frac{dy}{dx}$$

$$2xy + x^2\frac{dy}{dx} = \frac{dy}{dx}$$

$$x^2\frac{dy}{dx} - \frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx}(x^2-1) = -2xy$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy}{x^2-1}}$$

ex: $\sin x + \overbrace{2\cos}^{\text{out}}(\overset{\text{in}}{2y}) = 1$

$$\cos x \frac{dx}{dx} + 2\frac{dy}{dx} \cdot -2\sin(2y) = 0$$

$$\cos x - 4\sin 2y \frac{dy}{dx} = 0$$

$$\frac{-4\sin 2y \frac{dy}{dx}}{-4\sin 2y} = \frac{-\cos x}{-4\sin 2y}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{4\sin 2y}}$$

ex: $\underbrace{x^2}\underbrace{\cos y} = \overset{\text{out}}{\sin}(\overset{\text{in}}{xy}) + 3$

$$\cos y \cdot 2x\frac{dx}{dx} + x^2 \cdot -\sin y \frac{dy}{dx} = \left(y \cdot 1\frac{dx}{dx} + x \cdot 1\frac{dy}{dx}\right) \cdot \cos(xy) + 0$$

$$2x\cos y - x^2 \sin y \frac{dy}{dx} = \left(y + x\frac{dy}{dx}\right)\cos(xy)$$

$$2x\cos y - x^2 \sin y \frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$$

$$-x^2 \sin y \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy) - 2x\cos y$$

$$\frac{dy}{dx}\left(-x^2 \sin y - x\cos(xy)\right) = y\cos(xy) - 2x\cos y$$

$$\boxed{\frac{dy}{dx} = \frac{y\cos(xy) - 2x\cos y}{-x^2 \sin y - x\cos(xy)}}$$

ex: Find $\left.\frac{dy}{dx}\right|_{(-3,4)}$.

$$x^2 + y^2 = 25$$

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

sub in pt now (easier algebra) →

$$2(-3) + 2(4)\frac{dy}{dx} = 0$$

$$-6 + 8\frac{dy}{dx} = 0$$

$$8\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{6}{8}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(-3,4)} = \frac{3}{4}}$$