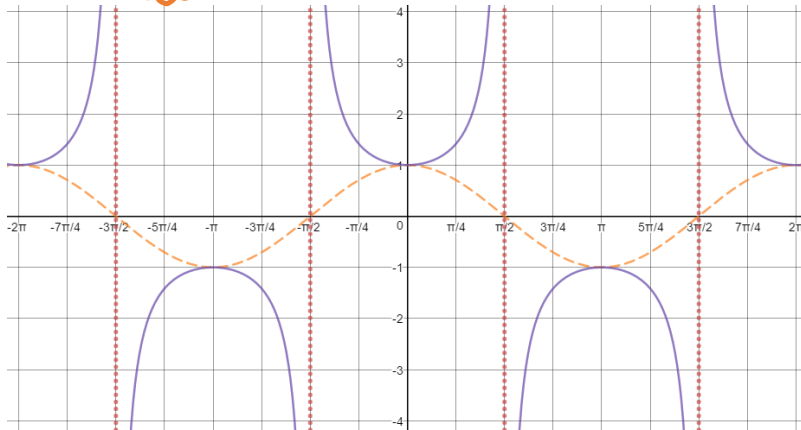## 4.5 Graphs of Tangent, Cotangent, Secant, Cosecant
Target 5E: Rigid and non-rigid transformations of sinusoids

$y = \sec x$ $\frac{1}{\cos x}$ 😊
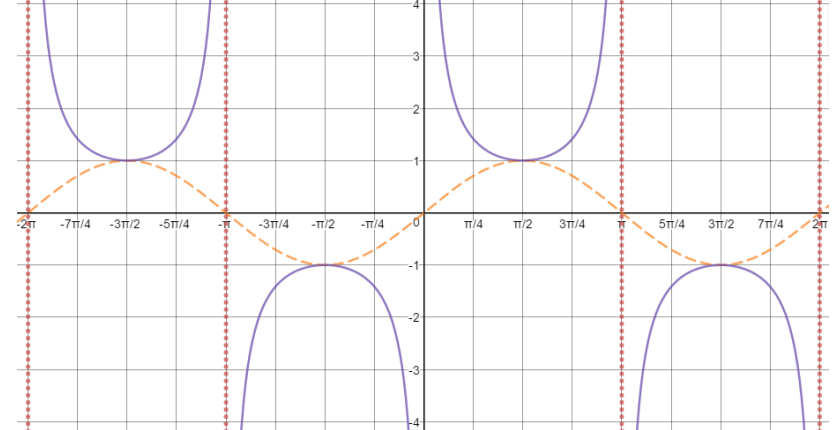

Period: __2π__

Asymptotes: $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots, \pm\frac{(2k+1)\pi}{2}$ where k is an integer

odd #s
2k+1
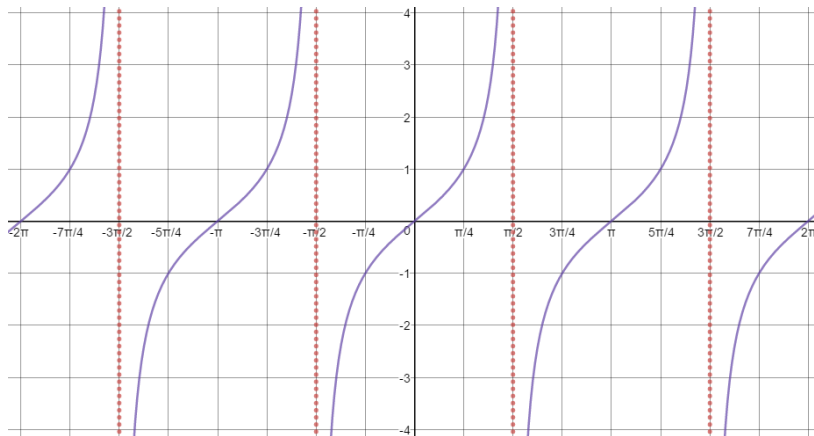*2(0)+1 = 1*
2(1)+1 = 3 😊

$y = \csc x$ $\frac{1}{\sin x}$ 😊

Period: __2π__

Asymptotes: $x = 0, \pm\pi, \pm2\pi, \cdots, 2k\pi$ where k is an integer

even #s
2k
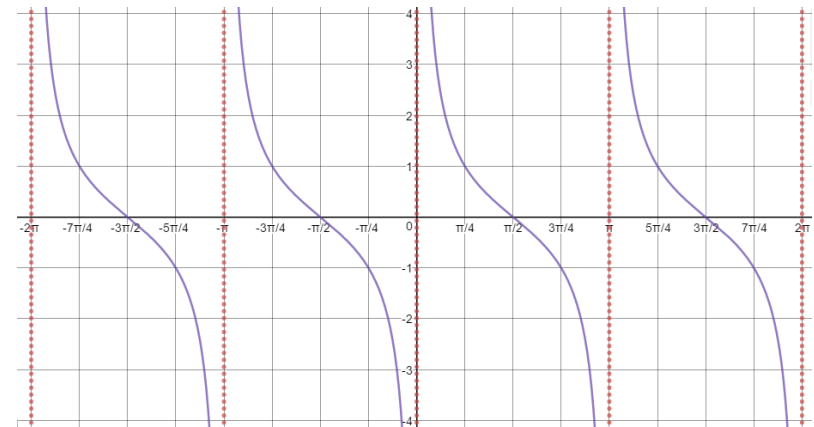*2(0) = 0*
2(1) = 2 😊

$y = \tan x$

Period: __π__

Asymptotes: $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots, \pm\frac{(2k+1)\pi}{2}$ where k is an integer

$y = \cot x$

Period: __π__

Asymptotes: $x = 0, \pm\pi, \pm2\pi, \cdots, 2k\pi$ where k is an integer

Write an equation for each graph and identify the vertical asymptotes.

1. Tangent

period $= 3\pi$

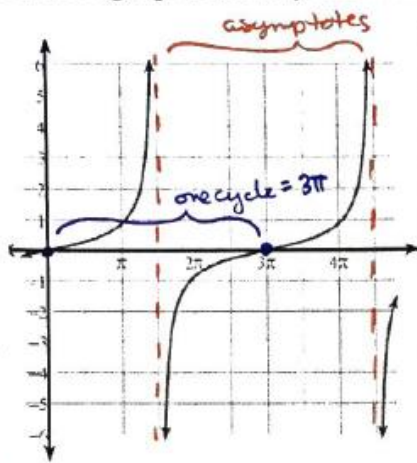$\frac{\pi}{b} = 3\pi$

$\pi = 3\pi b$

$\frac{1}{3} = b$

asymptotes

one cycle $= 3\pi$

$\boxed{y = \tan\left(\frac{1}{3}x\right)}$

V.A. @ $x = \frac{3\pi}{2}, \frac{9\pi}{2}, \frac{15\pi}{2}, \cdots$

$\boxed{x = \frac{3\pi}{2} + 3\pi k}$

(where k is an integer)

2. Cotangent

one cycle $= \frac{\pi}{2}$

period $= \frac{\pi}{2}$

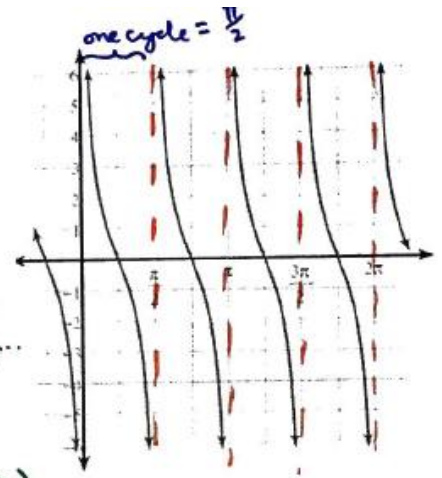$\frac{\pi}{b} = \frac{\pi}{2}$

$2\pi = \pi b$

$2 = b$

$\boxed{y = \cot(2x)}$

V.A. @ $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \cdots$

$\boxed{x = \frac{\pi}{2} k}$

(where k is an integer)

3. Secant

period $= \pi$

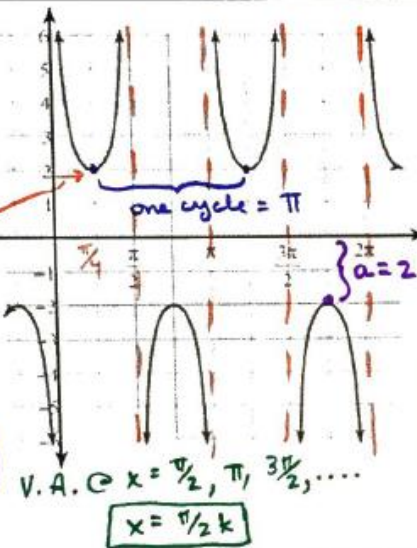$\frac{2\pi}{b} = \pi$

$2\pi = \pi b$

$2 = b$

phase shift $= \frac{\pi}{4}$

$\frac{c}{2} = \frac{\pi}{4}$

$c = \frac{\pi}{2}$

$a = 2$

$\boxed{y = 2\sec\left(2x + \frac{\pi}{2}\right)}$

one cycle $= \pi$

$a = 2$

V.A. @ $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \cdots$

$\boxed{x = \frac{\pi}{2} k}$

4. Cosecant

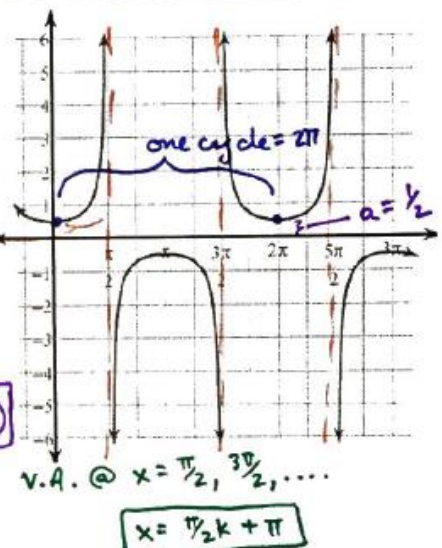period $= 2\pi$

$\frac{2\pi}{b} = 2\pi$

$b = 1$

phase shift $= \frac{\pi}{2}$

$\frac{c}{1} = \frac{\pi}{2}$

$c = \frac{\pi}{2}$

$a = \frac{1}{2}$

$\boxed{y = \frac{1}{2}\csc\left(x + \frac{\pi}{2}\right)}$

one cycle $= 2\pi$

$a = \frac{1}{2}$

V.A. @ $x = \frac{\pi}{2}, \frac{3\pi}{2}, \cdots$

$\boxed{x = \frac{\pi}{2} k + \pi}$

Describe the transformation for the given function.

5. $y = \tan(5x) + 2$

$\frac{\pi}{b} \rightarrow$ period

period $= \frac{\pi}{5}$

Vertical shift $= 2$ (up)

6. $y = \cot\left(\frac{x}{3} - \pi\right)$

period $= \frac{\pi}{b} = \frac{\pi}{\frac{1}{3}} = 3\pi$

phase shift $= \frac{c}{b} = \frac{\pi}{\frac{1}{3}} = 3\pi$ (to the right)

7. $y = -2\sec(3x)$

↓

reflects over x-axis

period $= \frac{2\pi}{b} = \frac{2\pi}{3}$

8. $y = \csc\left(2x + \frac{\pi}{2}\right)$

period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift $= \frac{c}{b} = \frac{\pi/2}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$ (to the left)

More Practice

Graphs of Secant, Cosecant, Tangent, and Cotangent
http://www.regentsprep.org/regents/math/algtrig/att7/othergraphs.htm
http://www.intmath.com/trigonometric-graphs/4-graphs-tangent-cotangent-secant-cosecant.php
http://www.purplemath.com/modules/triggrph3.htm
https://www.youtube.com/watch?v=2m_qvTv1RgU
https://www.youtube.com/watch?v=srWI_jFm91w
https://www.youtube.com/watch?v=4Fnu3_mXaik

Homework Assignment
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