AP Free-Response Questions

⊞**1.** The temperature outside a house during a 24-hour period is given by:

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), \quad 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

$$\text{avg temp} = \frac{1}{14-6} \int_6^{14} \left(80 - 10\cos\left(\frac{\pi t}{12}\right)\right) dt$$

$$= 87.162$$

$$= 87°F$$

2. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) What is the average number of gallons that leak out of the tank in the first three minutes?

$$\text{avg \# gallons} = \frac{1}{3-0} \int_0^3 \sqrt{t+1} \, dt$$

$$= \frac{1}{3} \int_0^3 (t+1)^{\frac{1}{2}} \, dt \qquad u = t+1$$

$$= \frac{1}{3} \int_1^4 u^{\frac{1}{2}} \, du \qquad \frac{du}{dt} = 1$$

$$du = dt$$
$$u(0) = 1$$

$$= \frac{1}{3}\left[\frac{2}{3} u^{3/2}\right]\Big|_1^4 \qquad u(3) = 4$$

$$= \frac{2}{9}\left[4^{3/2} - 1^{3/2}\right] = \frac{2}{9}(8-1) = \frac{14}{9} \text{ gallons}$$

⊞ 3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

$$\text{avg rate of water flow} = \frac{1}{24-0} \int_0^{24} \frac{1}{79}(768 + 23t - t^2)\, dt$$

$$= 10.785 \text{ gallons}/hr$$

⊞ 4. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the poind at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

$$\text{avg value of } P(t) = \frac{1}{15-0} \int_0^{15} P(t)\, dt$$

$$= \frac{1}{15} \int_0^{15} (20 + 10te^{-t/3})\, dt$$

$$= 25.757\,°C$$