6.1 Vectors in the Plane (continued)
Target 8A: Perform vector operations: scalar multiple and sums and represent them graphically
Target 8B: Perform vector operations: magnitude, direction angle, and unit vector
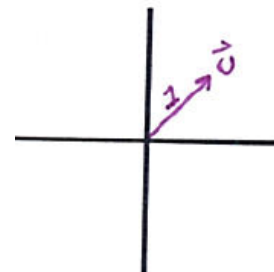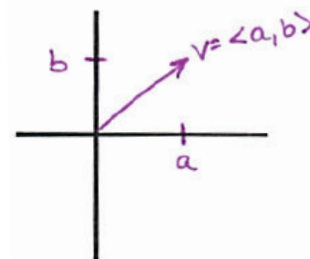
Unit Vector

Unit Vector – A vector, **u**, with a length of _one unit_ .

$$|u| = \underline{\ 1\ }$$

If **v** is not the zero vector, then the unit vector in the direction of **v** is:
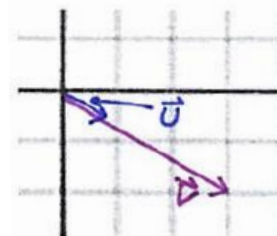
$$u = \frac{v}{|v|} = \frac{\langle v_1, v_2 \rangle}{\text{magnitude of } v}$$

The **Standard Unit Vectors** are $i = \langle 1,0 \rangle$ and $j = \langle 0,1 \rangle$.

So, any vector $v = \langle a, b \rangle$ can be expressed as

the linear combination of _ai + bj_ ,

where the scalar a is the _horizontal_ component

and b is the _vertical_ component

of vector **v**.

Example
Find a unit vector in the direction of $v = \langle 3, -2 \rangle$. Write the unit vector as a linear combination of the standard unit vector.

$$.\ .|v| = \sqrt{3^2 + (-2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

$$u = \frac{v}{|v|} = \frac{\langle 3, -2 \rangle}{\sqrt{13}}$$

$$\vec{u} = \langle \tfrac{3}{\sqrt{13}}, \tfrac{-2}{\sqrt{13}} \rangle$$

$$\vec{u} = \tfrac{3}{\sqrt{13}} i - \tfrac{2}{\sqrt{13}} j$$

A vector's **Direction Angle** is the angle θ that **v** makes with the positive *x*-axis.
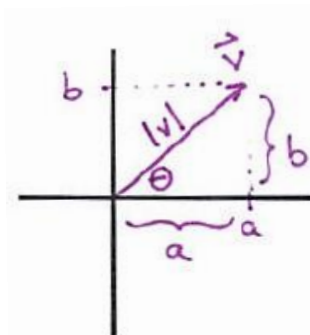
$$\sin \theta = \frac{b}{|v|} \qquad\qquad \cos \theta = \frac{a}{|v|}$$

So, $b = |v| \sin \theta$ \qquad\qquad So, $a = |v| \cos \theta$

The horizontal component is _$|v| \cos \theta$_ .

The vertical component is _$|v| \sin \theta$_ .

$$v = \underline{|v| \cos\theta\ i} + \underline{|v| \sin\theta\ j}$$

Example

Find the components of vector **v** with direction angle of 125° and magnitude of 8.

$$V = \langle 8\cos 125°, 8\sin 125°\rangle$$

$$V \approx \langle -4.589, 6.553\rangle$$
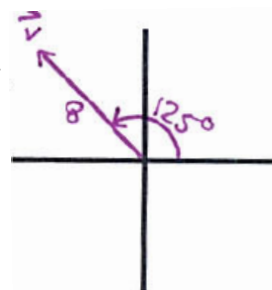
Practice

For #1 & 2, find the component form of the vector.

1.
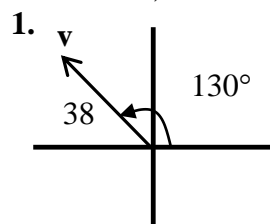
$$v = \langle 38\cos 130°, 38\sin 130°\rangle$$

$$v = \langle -24.426, 29.110\rangle$$

2.
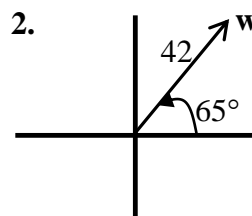
$$w = \langle 42\cos 65°, 42\sin 65°\rangle$$

$$w = \langle 17.75, 38.065\rangle$$

For #3-6, find:
   a) the component form of the vector,
   b) the unit vector in the direction of the given vector,
   c) and the direction angle of the vector.

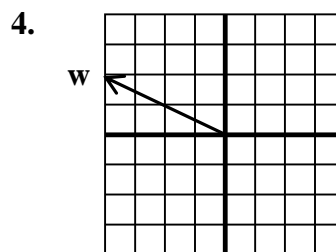**3.**
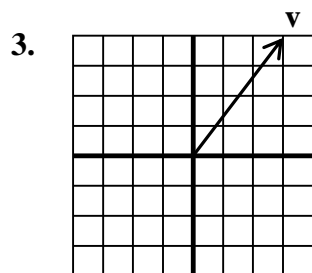
Component form

$$v = \langle 3, 4\rangle$$

$$|v| = \sqrt{3^2 + 4^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

$$u = \frac{v}{|v|} = \frac{\langle 3, 4\rangle}{5}$$

unit vector $$u = \langle \frac{3}{5}, \frac{4}{5}\rangle$$

$$\sin\theta = \frac{4}{5}$$
$$\theta = \sin^{-1}\left(\frac{4}{5}\right)$$
$$\theta = 53.130°$$

direction $$\theta \approx 53°$$

4.

Component form

$$w = \langle -4, 2\rangle$$

$$|w| = \sqrt{(-4)^2 + (2)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

$$u = \frac{w}{|w|} = \frac{\langle -4, 2\rangle}{2\sqrt{5}}$$

unit vector $$u = \langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$$

$$\cos\theta = \frac{-2}{\sqrt{5}}$$
$$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{5}}\right)$$
$$\theta = 153.435$$

direction $$\theta \approx 153°$$

5. $v = \langle -2, 3 \rangle$

$\boxed{v = \langle -2, 3 \rangle}$ component form

$|v| = \sqrt{(-2)^2 + 3^2}$
$= \sqrt{4 + 9}$
$= \sqrt{13}$

$u = \dfrac{v}{|v|} = \dfrac{\langle -2, 3 \rangle}{\sqrt{13}}$

unit vector $\boxed{u = \langle \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \rangle}$

$\cos\theta = \frac{-2}{\sqrt{13}}$
$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{13}}\right)$

$\theta = 123.690$
$\boxed{\theta \approx 124°}$ direction $\angle$

6. $w = 5i + 2j$

$\boxed{w = \langle 5, 2 \rangle}$ component form

$|w| = \sqrt{5^2 + 2^2}$
$= \sqrt{25 + 4}$
$= \sqrt{29}$

$u = \dfrac{\langle 5, 2 \rangle}{\sqrt{29}}$

$\boxed{u = \langle \frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}} \rangle}$ unit vector

$\cos\theta = \frac{5}{\sqrt{29}}$
$\theta = \cos^{-1}\left(\frac{5}{\sqrt{29}}\right)$

$\theta = 21.801$
$\boxed{\theta \approx 22°}$ direction angle

More Practice

Unit Vectors
https://www.khanacademy.org/math/precalculus/vectors-precalc#unit-vectors
https://www.mathsisfun.com/algebra/vector-unit.html
http://www.algebralab.org/lessons/lesson.aspx?file=Trigonometry_TrigVectorUnits.xml
http://www.softschools.com/formulas/physics/unit_vector_formula/83/
https://youtu.be/6o_S7u7Ddx4
https://youtu.be/iAeKl5h2SJM

Magnitude and Direction Angle of Vectors
https://braingenie.ck12.org/skills/108146
https://www.khanacademy.org/math/precalculus/vectors-precalc#magnitude-direction
https://www.varsitytutors.com/hotmath/hotmath_help/topics/magnitude-and-direction-of-vectors
http://www.softschools.com/math/pre_calculus/direction_angles_of_vectors/
https://youtu.be/WxWJorOVIj8
https://youtu.be/8Eur16foTMw
https://youtu.be/82nu_sAPmmo

Component Form of Vectors
https://www.khanacademy.org/math/precalculus/vectors-precalc#component-form-of-vectors
https://www.math10.com/en/geometry/vectors-operations/vectors-operations.html
https://www.varsitytutors.com/precalculus-help/express-a-vector-in-component-form
https://youtu.be/GxBUbiNL1eE
https://youtu.be/WZ3xzVHT0mc

Homework Assignment
p.511 #21-39odd