Solving Systems with Matrices Worksheet _____

Show all work for full credit. _____ **Date** _____

Find the inverse of each matrix, if it exists. No calculator.

1) $\begin{bmatrix} -2 & 6 \\ -1 & 3 \end{bmatrix}$

$\begin{vmatrix} -2 & 6 \\ -1 & 3 \end{vmatrix} = -2(3) - (-1)(6)$

$= -6 + 6$

$= 0$

∴, inverse DNE

2) $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

$\frac{1}{4(1) - 3(1)} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

$\frac{1}{4-3} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

3) $\begin{bmatrix} 14 & 8 \\ 6 & 4 \end{bmatrix}$

$\frac{1}{14(4) - 6(8)} \begin{bmatrix} 4 & -8 \\ -6 & 14 \end{bmatrix}$

$\frac{1}{56 - 48} \begin{bmatrix} 4 & -8 \\ -6 & 14 \end{bmatrix}$

$\frac{1}{8} \begin{bmatrix} 4 & -8 \\ -6 & 14 \end{bmatrix}$

$\begin{bmatrix} 1/2 & -1 \\ -3/4 & 7/4 \end{bmatrix}$

Solve for *X*. 2x2 by hand, 3x3 with calculator.

4) $\begin{bmatrix} 5 & 4 \\ -3 & 2 \end{bmatrix} X = \begin{bmatrix} 10 \\ -16 \end{bmatrix}$

$X = \begin{bmatrix} 5 & 4 \\ -3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ -16 \end{bmatrix}$

$X = \frac{1}{5(2) - (-3)(4)} \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ -16 \end{bmatrix}$

$X = \frac{1}{22} \begin{bmatrix} 2(10) + (-4)(-16) \\ 3(10) + 5(-16) \end{bmatrix}$

$X = \frac{1}{22} \begin{bmatrix} 84 \\ -50 \end{bmatrix} \rightarrow X = \begin{bmatrix} 42/11 \\ -25/11 \end{bmatrix}$

5) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 4 & 2 \\ 3 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} -5 \\ 15 \\ -7 \end{bmatrix}$

$X = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 4 & 2 \\ 3 & -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -5 \\ 15 \\ -7 \end{bmatrix}$

$X = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

Solve the following systems using matrices. Check your answers. All 2x2 systems must be done by hand. Systems 3x3 and larger may be solved with a calculator. However, show the matrix equation for all systems.

6) $\begin{cases} 3x - 7y = 7 \\ 7x + 3y = 3 \end{cases}$

$\begin{bmatrix} 3 & -7 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 7 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$= \frac{1}{3(3) - (-7)(7)} \begin{bmatrix} 3 & 7 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$= \frac{1}{58} \begin{bmatrix} 3(7) + 7(3) \\ -7(7) + 3(3) \end{bmatrix}$

$= \frac{1}{58} \begin{bmatrix} 42 \\ -40 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21/29 \\ -20/29 \end{bmatrix}$

$x = 21/29$, $y = -20/29$

7) $\begin{cases} x + 2y = 5 \\ 2x - 5y = -8 \end{cases}$

$\begin{bmatrix} 1 & 2 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1(-5) - 2(2)} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$= \frac{1}{-9} \begin{bmatrix} 5(5) + -2(8) \\ -2(5) + 1(8) \end{bmatrix}$

$= -\frac{1}{9} \begin{bmatrix} 9 \\ -2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2/9 \end{bmatrix}$

$x = -1$, $y = 2/9$

8) $\begin{cases} x+5y-10z=13 \\ 2x-y+3z=18 \\ -4x+6y+12z=7 \end{cases}$

$\begin{bmatrix} 1 & 5 & -10 \\ 2 & -1 & 3 \\ -4 & 6 & 12 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 18 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 5 & -10 \\ 2 & -1 & 3 \\ -4 & 6 & 12 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ 18 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8.672 \\ 3.907 \\ 1.521 \end{bmatrix}$

$x = 8.672,\ y = 3.907,\ z = 1.521$

9) $\begin{cases} -x+z=6 \\ 4y+3z=-1 \\ x-y=0 \end{cases}$

$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 4 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 4 & 3 \\ 1 & -1 & 0 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2.714 \\ -2.714 \\ 3.286 \end{bmatrix}$

$x = -2.714,\ y = -2.714,\ z = 3.286$

10) A flower farmer wants to plant three types of bulbs: gladiolas, irises, and tulips. The gladiolas cost \$75 per acre to plant, the irises cost \$100 per acre, and the tulips cost \$50 per acre.

a) The farmer wants to plant 200 acres of bulbs and spend a total of \$15,000. The farmer decides to plant twice as many gladiolas as irises. Write a system of equations and use a matrix to find the total number of acres of each type of flower.

$x+y+z=200$

$\$75x+\$100y+\$50z=\$15000$

gladiolas = x
irises = y
tulips = z

$x = 2y$

$\begin{cases} x-2y=0 \\ x+y+z=200 \\ 75x+100y+50z=15000 \end{cases}$

$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \\ 75 & 100 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 200 \\ 15000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \\ 75 & 100 & 50 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 200 \\ 15000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \\ 50 \end{bmatrix}$

100 gladiolas, 50 irises, and 50 tulips

b) The farmer decides to plant 250 acres instead, using the same amount of money. The farmer wants to keep the ratio of gladiolas to irises the same as in part a. Write the matrix equation and find the total number of acres planted of each type of flower.

$x+y+z=250$

$\begin{cases} x-2y=0 \\ x+y+z=250 \\ 75x+100y+50z=15000 \end{cases}$

$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \\ 75 & 100 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 250 \\ 15000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \\ 75 & 100 & 50 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 250 \\ 15000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 50 \\ 25 \\ 175 \end{bmatrix}$

50 gladiolas, 25 irises, and 175 tulips

11) Three companies produce three products – tables, lamps, and chairs. In one hour, the first company produces 1 table, 3 lamps, and 2 chairs. In the same amount of time, the second company produces 2 tables, 1 lamp, and 1 chair. The hourly production of the third company is 2 tables, 3 lamps, and 2 chairs.

a) Find the number of hours that each company needs to operate so their combined production fills an order for 60 tables, 78 lamps, and 56 chairs.

hours of 1ST company → x
of 2nd company → y
of 3rd company → z

$x + 2y + 2z = 60$
$3x + y + 3z = 78$
$2x + y + 2z = 56$

$$\begin{cases} x + 2y + 2z = 60 \\ 3x + y + 3z = 78 \\ 2x + y + 2z = 56 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 78 \\ 56 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 60 \\ 78 \\ 56 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 14 \end{bmatrix}$$

hours of 1ST company is 8,
2nd company is 12 hours,
3rd company is 14 hours

b) Find the number of hours that each company needs to operate so their combined production fills an order for 42 tables, 52 lamps, and 39 chairs.

$$\begin{cases} x + 2y + 2z = 42 \\ 3x + y + 3z = 52 \\ 2x + y + 2z = 39 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 42 \\ 52 \\ 39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 42 \\ 52 \\ 39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \\ 3 \end{bmatrix}$$

# of hours of 1ST company is 10,
2nd company is 13 hours,
3rd company is 3 hours