7.4 Partial Fractions

Target 8G: Decompose rational expressions into partial fractions

Review of Prior Concepts

More Practice

Simplifying Rational Expressions
http://www.mathplanet.com/education/algebra-1/rational-expressions/add-and-subtract-rational-expressions
http://www.purplemath.com/modules/rtnladd.htm
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions#adding-and-subtracting-rational-expressions
https://youtu.be/XTZl7Kn6u4Y
https://youtu.be/y_DweTAEYWk

Partial Fraction Decomposition

To **decompose a fraction** – write one fraction as the sum/difference of 2 or more fractions.

Example:
Write the terms for partial fraction decomposition of: $\frac{x-3}{x^2+3x}$

① Factor the denominator.

② Write each factor in its own fraction
   w/numerator as function that is one
   degree less than denominator.

More Examples:
Write the terms for partial fraction decomposition of each:

1. $\frac{x-14}{x^2-4}$

2. $\frac{4x+4}{x^3+2x^2}$

3. $\dfrac{3x^2+4}{(x-3)(x+5)(x^2+1)}$

4. $\dfrac{5x^3-10x^2-5x-5}{(x^2+9)(x^2+4)}$ ← already factored:

already factored:

$$\dfrac{3x^2+4}{(x-3)(x+5)(x^2+1)} = \dfrac{A}{x-3} + \dfrac{B}{x+5} + \dfrac{Cx+D}{x^2+1}$$

degree 0 degree 1

degree 1　degree 2

$$\dfrac{5x^3-10x^2-5x-5}{(x^2+9)(x^2+4)} = \dfrac{Ax+B}{x^2+9} + \dfrac{Cx+D}{x^2+4}$$

degree 1

degree 2

Decompose the fraction

Example:

Decompose: $\dfrac{x-3}{x^2+3x}$

① Factor the denominator.

② Write each factor in its own fraction w/numerator as function that is one degree less than denominator.

③ Solve for the constants: A,B,C, etc.

④ Write original fraction as decomposed fractions (partial fractions)

① $\dfrac{x-3}{x(x+3)}$

② $\dfrac{x-3}{x(x+3)} = \dfrac{A}{x} + \dfrac{B}{x+3}$

③ $\dfrac{x-3}{x(x+3)} = \dfrac{A}{x}(x)(x+3) + \dfrac{B}{x+3}(x)(x+3)$

$$x-3 = A(x+3) + B(x)$$
$$x-3 = Ax + 3A + Bx$$
$$x-3 = (A+B)x + 3A$$

constants = constants
linear coeff = linear coeff
...☺

$A+B = 1$　$-3 = 3A$
$-1 + B = 1$　$-1 = A$
$B = 2$

④ $\boxed{\dfrac{x-3}{x^2+3x} = \dfrac{-1}{x} + \dfrac{2}{x+3}}$

More Examples:

Decompose each fraction.

1. $\frac{x-14}{x^2-4}$

2. $\frac{7x-7}{x^2-3x-10}$

① $\frac{x-14}{(x-2)(x+2)}$

② $\frac{x-14}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$

③ $\frac{(x-2)(x+2) \cdot (x-14)}{(x-2)(x+2)} = \frac{A(x-2)(x+2)}{x-2} + \frac{B(x-2)(x+2)}{x+2}$

$x - 14 = A(x+2) + B(x-2)$

$x - 14 = Ax + 2A + Bx - 2B$

$x - 14 = (A+B)x + 2A - 2B$

$A+B = 1 \qquad 2A - 2B = -14$

$\qquad\qquad\qquad A - B = -7$

$A + B = 1$
$A - B = -7$
$\overline{2A = -6}$
$A = -3$

$A + B = 1$
$-3 + B = 1$
$B = 4$

$$\boxed{\frac{x-14}{x^2-4} = \frac{-3}{x-2} + \frac{4}{x+2}}$$

① $\frac{7x-7}{(x-5)(x+2)}$

② $\frac{7x-7}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$

③ $\frac{(x-5)(x+2) \cdot (7x-7)}{(x-5)(x+2)} = \frac{A(x-5)(x+2)}{x-5} + \frac{B(x-5)(x+2)}{x+2}$

$7x - 7 = A(x+2) + B(x-5)$

$7x - 7 = Ax + 2A + Bx - 5B$

$7x - 7 = (A+B)x + 2A - 5B$

$A + B = 7 \qquad 2A - 5B = -7$

$A = 7 - B \qquad 2(7-B) - 5B = -7$

$\qquad\qquad\qquad 14 - 2B - 5B = -7$

$A = 7 - 3 \qquad\qquad -7B = -21$

$A = 4 \qquad\qquad\qquad B = 3$

$$\boxed{\frac{7x-7}{x^2-3x-10} = \frac{4}{x-5} + \frac{3}{x+2}}$$

More Practice

Partial Fraction Decomposition

http://www.purplemath.com/modules/partfrac.htm

https://www.mathsisfun.com/algebra/partial-fractions.html

https://youtu.be/O14kJpRMY08

https://youtu.be/bqf42x6nZoo

Homework Assignment

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