One More Convergence Test

Alternating Series Test

The series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \cdots$$

converges if ALL three of the following conditions are true:

① each $u_n > 0$

② $u_n \geq u_{n+1} \ \forall \ n \geq N$ (where N is some integer)

③ $\lim_{n \to \infty} u_n = 0$

Determine if the series converges or diverges.

Example 1

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$$

Example 2

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

2 Types of Convergence

① Absolute convergence: If $\sum |a_n|$ converges, then $\sum a_n$ _____

Example:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

② Conditional convergence: If $\sum |a_n|$ diverges but $\sum a_n$ converges,

then $\sum a_n$ _____

Example:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

Determine if the series converges absolutely, converges conditionally, or diverges.

Example 1

$$\sum_{n=1}^{\infty} (-1)^n n^2 \left(\frac{2}{3}\right)^n$$

Example 2

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$