

Non-Calculator

① $\frac{dy}{dx} = 2(x^3+1)(3x^2)$ chain rule
 $= 6x^2(x^3+1)$

E

② $\int_0^1 e^{-4x} dx$

$u = -4x$
 $du = -4 dx$
 $\frac{du}{-4} = dx$

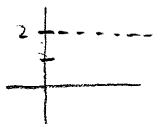
$u(0) = -4(0) = 0$
 $u(1) = -4(1) = -4$

↓

$= \int_0^{-4} e^u \cdot \frac{du}{-4}$
 $= -\frac{1}{4} \int_0^{-4} e^u du$
 $= -\frac{1}{4} e^u \Big|_0^{-4}$
 $= -\frac{1}{4} [e^{-4} - e^0]$
 $= -\frac{1}{4} (e^{-4} - 1) \Rightarrow -\frac{1}{4} e^{-4} + \frac{1}{4} \Rightarrow \frac{1}{4} - \frac{e^{-4}}{4}$

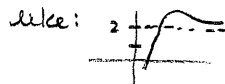
D

③ horizontal asympt. @ $y=2$, for $x \geq 0$



a) $f(0) = 2$, false b/c don't know y -value @ $x=0$

b) $f(x) \neq 2$, for all $x \geq 0$ false, graph might look



c) $f(2)$ undefined, false don't know when $x=2$ what the y -value is.

d) $\lim_{x \rightarrow 2} f(x) = \infty$, false graph will follow asymptote not keep going up to ∞ .

E e) $\lim_{x \rightarrow \infty} f(x) = 2$, true as x goes to infinity, the graph goes to 2.

④ $\frac{dy}{dx} = \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2}$ quotient rule.

$= \frac{6x+4 - 6x-9}{(3x+2)^2}$
 $= \frac{-5}{(3x+2)^2}$

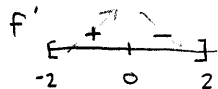
D

⑤ $\int_0^{\pi/4} \sin x \, dx$
 $= -\cos x \Big|_0^{\pi/4}$
 $= -\cos \pi/4 - (-\cos 0)$
 $= -\frac{\sqrt{2}}{2} + 1$ D

⑥ $\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} = \frac{1}{4}$
 same power, so take coefficients.

or
 $\lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^3} - \frac{2x^2}{x^3} + \frac{3x}{x^3} - \frac{4}{x^3}}{\frac{4x^3}{x^3} - \frac{3x^2}{x^3} + \frac{2x}{x^3} - \frac{1}{x^3}}$ divide by highest power.
 $= \lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x} + \frac{3}{x^2} - \frac{4}{x^3}}{4 - \frac{3}{x} + \frac{2}{x^2} - \frac{1}{x^3}} \rightarrow 0$
 $= \frac{1}{4}$ C

⑦ Looking at graph of derivative \rightarrow look where above & below x-axis to get f.



f inc (-2, 0), f dec. (0, 2)

- (A) false - inc & dec.
- (B) true B
- (C) false - dec.
- (D) false - max
- (E) false b/c f' has y-values

⑧ $\int x^2 \cos(x^3) \, dx$

$u = x^3$
 $du = 3x^2 \, dx$
 $\frac{du}{3x^2} = dx$

$\int x^2 \cos u \cdot \frac{du}{3x^2}$
 $= \frac{1}{3} \int \cos u \, du$
 $= \frac{1}{3} \sin u + C$
 $= \frac{1}{3} \sin(x^3) + C$ B

⑨ $f(x) = \ln(x + 4 + e^{-3x})$
 $f'(x) = \frac{1}{x + 4 + e^{-3x}} \cdot (1 + e^{-3x}(-3))$
 $f'(0) = \frac{1}{0 + 4 + e^0} \cdot (1 + e^0(-3))$
 $= \frac{1}{4 + 1} (1 - 3)$
 $= \frac{-2}{5}$ A

⑩ $f(x) < 0 \rightarrow$ all pts below x-axis
 $f'(x) < 0 \rightarrow$ decreasing
 $f''(x) < 0 \rightarrow$ concave down.

- (A) no, inc & dec.
- (B) yes B
- (C) no, inc
- (D) no, conc. up & down
- (E) no, pts above x-axis

⑪ $u = 2x + 1, \int_0^2 \sqrt{2x + 1} \, dx$

\downarrow
 $= \int_1^5 \sqrt{u} \cdot \frac{du}{2}$
 $= \frac{1}{2} \int_1^5 \sqrt{u} \, du$ C

$u = 2x + 1$
 $du = 2 \, dx$
 $\frac{du}{2} = dx$

$u(0) = 2(0) + 1 = 1$
 $u(2) = 2(2) + 1 = 5$

12) rate of change of volume w/ respect to time $\rightarrow \frac{dV}{dt}$

directly proportional $\rightarrow k \cdot \sqrt{V}$
to sq. root of volume

$$\frac{dV}{dt} = k\sqrt{V} \quad \boxed{E}$$

13) $f \rightarrow$ cont; no holes, jumps, asymptotes.

$f \rightarrow$ diff'able; no jumps, asymptotes, sharp turns, vertical tangents

(A) $a \rightarrow$ cont, but sharp turn

\boxed{A}

(B) $b \rightarrow$ not cont. (hole)

(C) $c \rightarrow$ cont, diff'able

(D) $d \rightarrow$ not cont (jump)

(E) $e \rightarrow$ cont, diff'able

14) $\frac{dy}{dx} = x^2(\cos 2x) \cdot 2 + \sin 2x \cdot 2x$ product rule.

$$= 2x^2 \cos 2x + 2x \sin 2x$$

$$= 2x(x \cos 2x + \sin 2x) \quad \boxed{E}$$

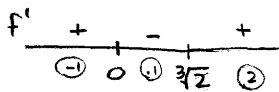
15) $f'(x) = x^2 - \frac{2}{x}$

$$0 = x^2 - \frac{2}{x}, \quad x \neq 0$$

$$\frac{2}{x} = x^2$$

$$x^3 = 2$$

$$x = \sqrt[3]{2}$$



dec. $(0, \sqrt[3]{2}]$ \boxed{D}

16) $f'(1) = \frac{f(1) - f(-2)}{1 - (-2)}$ slope!

$$= \frac{7 - (-2)}{1 - (-2)}$$

$$= \frac{9}{3}$$

$$= 3 \quad \boxed{C}$$

17) $f(x) = 2xe^x$ concavity $\rightarrow f''(x)!$

$f'(x) = 2xe^x + e^x \cdot 2$ product rule

$$= 2e^x(x+1)$$

$$f''(x) = 2e^x(1) + (x+1)(2e^x)$$

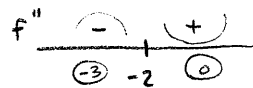
$$= 2e^x(1+x+1)$$

$$= 2e^x(x+2)$$

$$2e^x(x+2) = 0$$

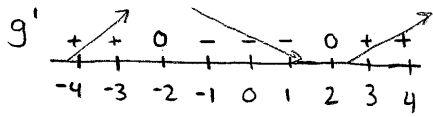
$$2e^x \neq 0 \quad x+2=0$$

$$x = -2$$



conc. down $(-\infty, -2)$
or $x < -2$ \boxed{A}

18



g decreasing $-2 \leq x \leq 2$ **A**

19

slope $\rightarrow 2x + 3$

original equation?

$$\int 2x + 3 \, dx$$

$$y = x^2 + 3x + C$$

check C, or D using pt. (1,2)

(C) $2 = 1^2 + 3(1)$

$$2 = 1 + 3$$

$$2 \neq 4$$

(D) $2 = 1^2 + 3(1) - 2$

$$2 = 1 + 3 - 2$$

$$2 = 2 \checkmark$$

D

20

I. $\lim_{x \rightarrow 3} f(x)$ exists \rightarrow true if $\lim_{x \rightarrow 3^-} = \lim_{x \rightarrow 3^+}$

$$\begin{aligned} \lim_{x \rightarrow 3^-} x + 2 &= \lim_{x \rightarrow 3^+} 4x - 7 \\ 3 + 2 &= 4(3) - 7 \\ 5 &= 5 \end{aligned}$$

D I + II only!

II. f cont. @ $x=3 \rightarrow$ true if $\lim_{x \rightarrow 3} f(x) = f(3)$

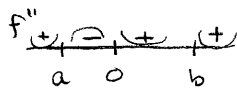
$$\begin{aligned} 5 &= 3 + 2 \\ 5 &= 5 \end{aligned}$$

III. f diff'able @ $x=3$, true if $f'_{left}(3) = f'_{right}(3)$

$$\begin{aligned} 1 &\neq 4 \text{ false.} \end{aligned}$$

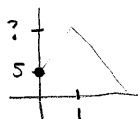
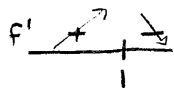
21

looking at 2nd derivative. - concavity
pt. of inf \rightarrow when f changes concavity



pt. of inf @ a and 0. **A**

22



(a) too small

(b) too small

(c) 6 \rightarrow if $f'(x) = 0$ $0 \leq x \leq 1$; it doesn't!

(d) 8 \rightarrow since $f'(x) > 0$ for $0 \leq x \leq 1$

(e) 11 \rightarrow if $f'(x) = 6$, $0 \leq x \leq 1$; it doesn't!

OR

$$f'(x) = -6x + 6$$

$$f(x) = -3x^2 + 6x + C$$

$$f(0) = -3(0)^2 + 6(0) + C$$

$$5 = C$$

D

$$f(x) = -3x^2 + 6x + 5$$

$$f(1) = -3 + 6 + 5$$

$$f(1) = 8$$

23 $\frac{d}{dx} \int_0^{x^2} \sin(t^3) dt$

by 2nd FTC, \swarrow chain rule

$$= \sin(x^2)^3 \cdot 2x$$

$$= 2x \sin(x^6) \quad \boxed{E}$$

24 $f(x) = 4x^3 - 5x + 3$ tangent line - need slope + pt.

$$f'(x) = 12x^2 - 5$$

$$f'(-1) = 12(-1)^2 - 5$$

$$= 12 - 5$$

$$= 7 \rightarrow \text{slope}$$

$$f(-1) = 4(-1)^3 - 5(-1) + 3$$

$$= -4 + 5 + 3$$

$$= 4 \rightarrow \text{pt. } (-1, 4)$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 7(x + 1)$$

$$y = 7x + 11 \quad \boxed{C}$$

25 $x(t) = 2t^3 - 2t^2 + 72t - 53$ position

$x'(t) = v(t) \rightarrow$ velocity when $v(t) = 0$, particle is resting.

$$v(t) = 6t^2 - 42t + 72$$

$$0 = 6(t^2 - 7t + 12)$$

$$0 = 6(t-3)(t-4)$$

$$6=0 \quad t-3=0 \quad t-4=0$$

$$t=3 \quad t=4 \quad \boxed{E}$$

26 slope \rightarrow derivative!

$$3y^2 - 2x^2 = 6 - 2xy \quad @ (3, 2)$$

$$6y \frac{dy}{dx} - 4x = -2x \frac{dy}{dx} + y(-2)$$

$$6(2) \frac{dy}{dx} - 4(3) = -2(3) \frac{dy}{dx} + (2)(-2)$$

$$12 \frac{dy}{dx} - 12 = -6 \frac{dy}{dx} - 4$$

$$18 \frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \frac{8}{18} \rightarrow \frac{4}{9} \quad \boxed{B}$$

27 $f(x) = x^3 + x$

$$f'(x) = 3x^2 + 1$$

$$f'(1) = 3 + 1 = 4$$

$$g(x) = f^{-1}(x)$$

\downarrow

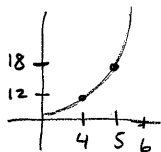
$$g'(x) = \frac{1}{f'(y)}$$

$$g'(2) = \frac{1}{f'(1)}$$

$$g'(2) = \frac{1}{4} \quad \boxed{B}$$

28 $g'(x) > 0 \rightarrow g$ increasing

$g''(x) > 0 \rightarrow g$ concave up



a) 15 too small

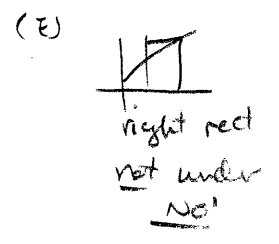
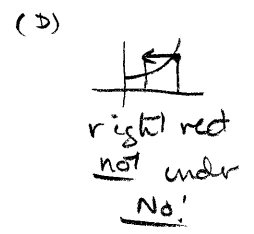
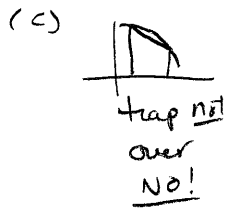
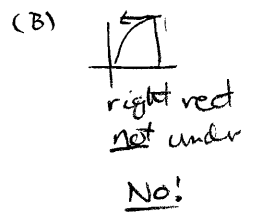
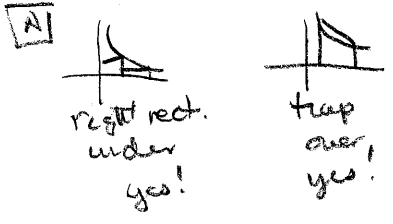
b) 18 too small

c) 21 inc but not high enough

d) 24 inc by 6, would make a straight line

e) 27 \checkmark \boxed{E}

29 trapezoid goes over, right rectangles under



30

$$\frac{1}{4-2} \int_2^4 f(t) dt = 1$$

$$\frac{1}{2} \int_2^4 f(t) dt = 1$$

$$\int_2^4 f(t) dt = 2$$

need area from $x=2$ to $x=4$ to be 2.

