2008 AP Calculus AB

Section I Answer Key and Percent Answering Correctly
Calculus AB

Item No.	Correct Answer	Percent Correct by Grade 5	4	3	2	1	Total Percent Correct
1	B	88	77	68	58	36	65
2	D	96	84	71	57	32	67
3	D	72	48	35	26	14	39
4	B	95	86	74	60	37	70
5	A	74	59	49	40	24	49
6	A	74	55	46	39	30	49
7	B	97	89	76	55	22	67
8	E	86	74	64	52	29	61
9	D	95	82	65	43	16	60
10	C	90	79	69	58	35	66
11	B	98	92	82	69	44	77
12	D	89	79	69	57	33	65
13	A	72	51	36	24	14	40
14	E	50	30	25	23	18	29
15	C	93	82	72	61	41	70
16	D	74	46	30	20	11	37
17	C	86	62	43	31	26	50
18	A	47	23	15	12	11	22
19*	–	–	–	–	–	–	–
20	D	73	50	35	25	19	41
21	A	48	26	17	11	7	22
22	B	80	64	58	55	50	62
23	E	50	26	17	12	7	23
24	B	80	54	33	18	9	40
25	B	55	26	16	14	13	26
26	A	53	29	17	12	9	25
27	C	70	52	41	31	20	43
28	A	38	13	6	3	3	14

Part A—No Calculator (Average number correct = 12.8)

Question #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Correct/Incorrect														
Percent of Students Answering Correctly	*65*	*67*	*39*	*70*	*49*	*49*	*67*	*61*	*60*	*66*	*77*	*65*	*40*	*29*

Question #	15	16	17	18	20	21	22	23	24	25	26	27	28
Correct/Incorrect													
Percent of Students Answering Correctly	*70*	*37*	*50*	*22*	*41*	*22*	*62*	*23*	*40*	*26*	*25*	*43*	*14*

2008 M/C Non-Calculator

(1) $\lim_{x \to \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)} = \frac{-2x^2}{x^2}$

$= -2$

B

(2) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

$= -x^{-1} + C$

D

(3) $f(x) = (x-1)(x^2+2)^3$

$f'(x) = (x-1)\left[3(x^2+2)^2(2x)\right] + (x^2+2)^3(1)$

$= (x^2+2)^2\left[(x-1)6x + (x^2+2)\right]$

$= (x^2+2)^2(6x^2 - 6x + x^2 + 2)$

$= (x^2+2)^2(7x^2 - 6x + 2)$

D

(4) $\int \sin(2x)\,dx + \int \cos(2x)\,dx$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\int \sin u \cdot \frac{du}{2} + \int \cos u \cdot \frac{du}{2}$

$\frac{1}{2}(-\cos u) + \frac{1}{2}\sin u + C$

$= -\frac{1}{2}\cos 2x + \frac{1}{2}\sin 2x + C$

B

(5) $\lim_{x \to 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2} = \lim_{x \to 0} \frac{x^2(5x^2+8)}{x^2(3x^2-16)}$

$= \lim_{x \to 0} \frac{5x^2+8}{3x^2-16}$

$= \frac{8}{-16}$

$= -\frac{1}{2}$

A

(6) I $\lim_{x \to 2} f$

II $f(2) = 1$

$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2}$

$= 2+2$

$= 4$

not cont.

III. not diff'able b/c not cont.

A

(7) $v(t) = 3t^2 + 6t$ $\quad s(0) = 2$ $\quad s(1) = ?$

$s(t) = \int (3t^2 + 6t)\,dt$

$s(t) = t^3 + 3t^2 + C$

$s(0) = 0^3 + 3(0)^2 + C$

$2 = 0 + 0 + C$

$2 = C$

$s(1) = 1^3 + 3(1)^2 + 2$

$= 1 + 3 + 2$

$= 6$

B

(8) $f(x) = \cos(3x)$

$f'(x) = -\sin(3x) \cdot 3$

$= -3\sin(3x)$

$f'\left(\frac{\pi}{9}\right) = -3\sin\left(3 \cdot \frac{\pi}{9}\right)$

$= -3\sin\left(\frac{\pi}{3}\right)$

$= -3\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{3\sqrt{3}}{2}$

E

(9) $g(x) = \int_{-2}^{x} f(t)\,dt$ → area

A. neg. area.

B. 0

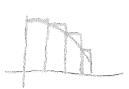
C. ▨ → $\frac{1}{2}(1+2)(2) = 3$

D. ▨ + △ ⇒ $3 + \frac{1}{2}(1)(2) = 4$

E. same as C.

(10) A.

B.



over

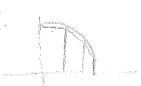
C

under.

D.

over & under

E.



under

(11)

B

(12) $f(x) = e^{2/x}$

$= e^{2x^{-1}}$

$f'(x) = e^{2x^{-1}} \cdot -2x^{-2}$

D

(13) $f(x) = x^2 + 2x$

$f'(x) = 2x + 2$

$\frac{d}{dx}(f(\ln x)) = f'(\ln x) \cdot \frac{1}{x}$

$= [2(\ln x) + 2] \frac{1}{x}$

A

(14)

f" changes signs on interval (0,2) so f changes concavity

E

not necessarily D, b/c don't know what f" on (0,1) or (1,2) is.

could be:

x	0	0.5	1	1.2	2
f"	5	↑ -#	0	↑ -#	-7

f"(0) = 5 doesn't mean all f" on (0,1) are positive

(15) $\int \frac{x}{x^2-4}\,dx$ $\quad u = x^2 - 4$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \frac{x}{u} \cdot \frac{du}{2x}$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2-4| + C$

C

(16) $\sin(xy) = x$

$\cos(xy) \cdot \left[y \cdot 1 + x\frac{dy}{dx}\right] = 1$

$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = 1$

$x\cos(xy)\frac{dy}{dx} = 1 - y\cos(xy)$

$\frac{dy}{dx} = \frac{1 - y\cos(xy)}{x\cos(xy)}$

D

(17) $g(x) = \int_0^x f(t)\,dt$

$g'(x) = \frac{d}{dx}\int_0^y f(t)\,dt$

$= f(x)$

$g''(x) = f'(x)$

$g''(x) = 0$ when $f'(x) = 0$

@ $x = 2, x = 5$

$g'' = f'$ + − + (2, 5)

inf pt @ x = 2, 5 b/c g" changes signs

C

(18) $y = -x + k \rightarrow$ tangent line

$y = x^2 + 3x + 1$

$y' = 2x + 3$

$m = m$

$-1 = 2x + 3$

$-4 = 2x$

$x = -2$

$y = (-2)^2 + 3(-2) + 1$

$= 4 - 6 + 1$

$y = -1$

$y = -x + k$

$-1 = -(-2) + k$

$-1 = 2 + k$

$-3 = k$

A

(19) H.A. $\lim_{x\to\infty} \frac{5 + 2^x}{1 - 2^x} = -1$

$\lim_{x\to-\infty} \frac{5 + 2^x}{1 - 2^x}$

$= \frac{5 + 2^{-\infty}}{1 - 2^{-\infty}} = \frac{5 + \frac{1}{2^\infty}}{1 - \frac{1}{2^\infty}} = \frac{5}{1}$

$= 5$

E

(20) $f'' = x^2(x-3)(x-6)$

$x = 0, x = 3, x = 6$ $\quad f''$ + (−1) + (1) − (4) + (7)

inf. pt @ x = 3 & x = 6 b/c f' changes signs

(21) $x(t) \Rightarrow$ position

$v(t) \Rightarrow x'(t)$

$v(t)$ increasing $\Rightarrow x''(t)$ positive

so, conc. up on $(0,2)$

A

(22) $\frac{dP}{dt} = k(\#\text{people who heard})(\#\text{people not heard})$

$= k(p)(N-p)$

$N \Rightarrow$ total population

B

(23) $\frac{dy}{dx} = \frac{x^2}{y}$

$\int y\,dy = \int x^2\,dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C \qquad y(3) = -2$

$\frac{1}{2}(-2)^2 = \frac{1}{3}(3)^3 + C$

$2 = 9 + C$

$-7 = C$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 - 7$

$y^2 = \frac{2}{3}x^3 - 14$

$y = \pm\sqrt{\frac{2}{3}x^3 - 14}$

E b/c y neg.

(24) $f(2) = 1, \quad f'(2) = 4, \quad f''(2) = 3$

$f(1.9)$

$y - y_1 = m(x - x_1)$

$y - f(2) = f'(2)(1.9 - 2)$

$y - 1 = 4(-.1)$

$y - 1 = -.4$

$y = .6$

B

(25) cont.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^-} (cx+d) = \lim_{x\to 2^+} (x^2 - cx)$

$2c + d = 2^2 - c(2)$

$2c + d = 4 - 2c$

$4c + d = 4$

$4(2) + d = 4$

$8 + d = 4$

$d = -4$

diff'able

$f'(x) = \begin{cases} c & x \le 2 \\ 2x - c & x > 2 \end{cases}$

$\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$

$\lim_{x\to 2^-} c = \lim_{x\to 2^+} (2x - c)$

$c = 2(2) - c$

$2c = 4$

$c = 2$

$c + d = 2 + -4$

$= -2$ B

(26) $y = \arctan(4x)$

$y' = \frac{1}{1+(4x)^2} \cdot 4$

$y'(\frac{1}{4}) = \frac{1}{1+(4\cdot\frac{1}{4})^2} \cdot 4$

$= \frac{1}{1+1} \cdot 4$

$= \frac{1}{2} \cdot 4$

$= 2$

A

(27) $\frac{dy}{dx} = 0$ @ $x = -1$ and $y = 0$

(A) $\frac{dy}{dx} = (-1)(y) = -y \neq 0$ ×

(B) $\frac{dy}{dx} = -1y - y = -2y \neq 0$ ×

(C) $\frac{dy}{dx} = -1y + y = 0$ ✓

$\frac{dy}{dy} = -x(0) + 0 = 0$ ✓

(D) $\frac{dy}{dx} = -1y + (-1) \neq 0$ ×

(E) $\frac{dy}{dx}$ not dependent on y ×

(28) $g(x) = f^{-1}(x) \Rightarrow$ inverses

$f(6) = 3 \qquad g(3) = 6$

$f'(6) = -2 \qquad g'(3) = -\frac{1}{2}$

reciprocal

A