2012 | AB
NonCalculator

① $y = x\sin x$

$\frac{dy}{dx} = \sin x(1) + x(\cos x)$

$= \sin x + x\cos x$

$\boxed{B}$

② f inc $\to f' > 0$

$F(x) = 300x - x^3$

$f'(x) = 300 - 3x^2$

$0 = 3(100 - x^2)$

$100 - x^2 = 0$

$x^2 = 100$

$x = \pm 10$

f' − + −

(−11) −10 (0) 10 (11)

$\boxed{B}$ f inc on $(-10, 10)$

③ $\int \sec x \tan x \, dx$

$\boxed{A}$ $= \sec x + C$

④ $f(x) = 7x - 3 + \ln x$

$f'(x) = 7 + \frac{1}{x}$

$f'(1) = 7 + \frac{1}{1}$

$= 8$

$\boxed{E}$

⑤ $\lim_{x\to 2} f(x) = 2$ ✓

$\lim_{x\to 3} f(x) = 5$ ✓

$\lim_{x\to 4^-} f(x) = 2$, $\lim_{x\to 4^+} f(x) = 4$ $\}$ $\neq$ FALSE $\boxed{C}$

$\lim_{x\to 5} f(x) = 6$ ✓

$\lim_{x\to 3} f(x) = 5$, $f(3) = 5$ $\} =$ ✓

⑥ $v(t) = 6t - t^2$

total distance $= \int_0^3 |6t - t^2| \, dt$

$= \int_0^3 (6t - t^2) \, dt$

$= (3t^2 - \frac{1}{3}t^3)\Big|_0^3$

$= 3(3)^2 - \frac{1}{3}(3)^3 - 0$

$= 27 - 9$

$\boxed{D}$ $= 18$

⑦ $y = (x^3 - \cos x)^5$

$y' = 5(x^3 - \cos x)^4 (3x^2 + \sin x)$

$\boxed{E}$

⑧ oil in tank $=$ initial amount $+ \int_4^{15} R(t) \, dt$

$= 50 + 3(5.6) + 5(5.9) + 3(6.2)$

$= 50 + 16.8 + 29.5 + 18.6$

$= 50 + 35.4 + 29.5$

$\boxed{C}$ $= 50 + 64.9$

$= 114.9$

⑨ f cont $\to \lim_{x\to 2} f(x) = f(2)$

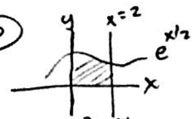
$\lim_{x\to 2} \frac{(2x+1)\cancel{(x-2)}}{\cancel{x-2}} = k$

$\lim_{x\to 2} (2x+1) = k$

$5 = k$

$\boxed{E}$

⑩



$u = x/2$

$du = \frac{1}{2} dx$

$2du = dx$

$u(2) = 1$

$u(0) = 0$

Area $= \int_0^2 e^{x/2} \, dx$

$= \int_0^1 e^u \cdot 2 \, du$

$= 2e^u \Big|_0^1$

$= 2(e^1 - e^0)$

$\boxed{A}$ $= 2e - 2$

⑪ $f(x) = \sqrt{|x-2|} = \begin{cases} \sqrt{x-2} & \text{for } x \geq 2 \\ \sqrt{-(x-2)} & \text{for } x < 2 \end{cases}$

$f'(x) = \begin{cases} \frac{1}{2}(x-2)^{-1/2} & x \geq 2 \\ -\frac{1}{2}[-(x-2)]^{-1/2} & x < 2 \end{cases} = \begin{cases} \frac{1}{2\sqrt{x-2}} & x \geq 2 \\ -\frac{1}{2\sqrt{-(x-2)}} & x < 2 \end{cases}$

$\lim_{x\to 2^-} f(x) = 0$, $f(2) = 0$ $\therefore$, f cont @ $x = 2$

$\lim_{x\to 2^+} f(x) = 0$

$\lim_{x\to 2^-} f'(x) =$ DNE, $\therefore$ f not diff'able @ $x = 2$

$\boxed{A}$

⑫ $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2} dx$

$2x^{1/2} du = dx$

$u(4) = \sqrt{4} = 2$

$u(1) = \sqrt{1} = 1$

$\int_1^2 \frac{e^u}{\cancel{\sqrt{x}}} \cdot 2\cancel{x^{1/2}} du$

$\boxed{C}$ $= 2\int_1^2 e^u \, du$

⑬ $f(x) = \begin{cases} 2 & x < 3 \\ x-1 & x \geq 3 \end{cases}$

$\int_1^5 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx$

$= \int_1^3 2\,dx + \int_3^5 (x-1)\,dx$

$= 2x\Big|_1^3 + (\frac{1}{2}x^2 - x)\Big|_3^5$

$= 2(3) - 2(1) + \frac{1}{2}(5)^2 - 5 - (\frac{1}{2}(3)^2 - 3)$

$= 4 + \frac{25}{2} - 5 - \frac{9}{2} + 3$

$= 2 + \frac{16}{2}$

$\boxed{D}$ $= 2 + 8 = 10$

⑭ derivative of $f(g(x))\Big|_{x=3}$

$f'(g(x)) \cdot g'(x)$

$f'(g(3)) \cdot g'(3)$

$f'(7) \cdot 3$

$\frac{7}{\sqrt{45}} \cdot 3$

$\frac{21}{\sqrt{45}} = \frac{21}{\sqrt{9 \cdot 5}}$

$= \frac{21}{3\sqrt{5}}$

$\boxed{A}$ $= \frac{7}{\sqrt{5}}$

$g(3) = 3(3) - 2 = 7$

$g'(x) = 3$

$g'(3) = 3$

$f'(x) = \frac{1}{2}(x^2-4)^{-1/2} \cdot 2x$

$= x(x^2-4)^{-1/2}$

$f'(7) = 7(7^2-4)^{-1/2}$

$= \frac{7}{\sqrt{45}}$

⑮ $h(6) = \int_0^6 f(t)\,dt$ area of $f(t)$ from 0 to 6

$h(6) < 0$

$h'(x) = \frac{d}{dx}\int_0^x f(t)\,dt$

$h'(x) = f(x)$

$h'(6) = f(6)$

$h'(6) = 0$

$h''(x) = f'(x)$

$h''(6) = f'(6)$

$h''(6) > 0$

$\boxed{A}$ $h(6) < h'(6) < h''(6)$

⑯ particle rest $\to v(t) = 0$

$x(t) = (t-a)(t-b)$

$v(t) = x'(t) = (t-b)(1) + (t-a)(1)$

$= t - b + t - a$

$0 = 2t - b - a$

$0 = 2t - b - a$

$b + a = 2t$

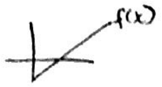
$\frac{b+a}{2} = t$

$\boxed{B}$

(17) $f(x) = \int_2^x g(t)\,dt$

$f'(x) = g(x)$

$f(x) \Rightarrow$



$f'(x) > 0$ and constant

$\therefore,\ g(x) \rightarrow$

$\boxed{A}$

(18) Definition of derivative

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(4) = \lim_{h \to 0} \frac{\ln(4+h) - \ln 4}{h}$

$f(x) = \ln x$ @ $x = 4$

$f'(x) = \frac{1}{x}$

$\boxed{B}$ $f'(4) = \frac{1}{4}$

(19) $f(x) = \frac{x}{x+2}$

$f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2}$

$= \frac{x+2-x}{(x+2)^2}$

$\frac{1}{2} = \frac{2}{(x+2)^2}$

$\sqrt{(x+2)^2} = \sqrt{4}$

$x+2 = \pm 2$

$x+2 = 2$ $\quad$ $x+2 = -2$

$x = 0$ $\quad$ $x = -4$

$\boxed{C}$ $f(0) = 0$ $\quad$ $f(-4) = \frac{-4}{-4+2} = \frac{-4}{-2} = 2$

$(0,0)$ $\quad$ $(-4,2)$

(20) $f(0) = 1$ $\quad$ $g(1) = 0$

$f'(0) = 6$ $\quad$ $g'(1) = \frac{1}{6}$

reciprocal

$f'(x) = 3(2x+1)^2(2)$

$f'(0) = 3(1)^2(2)$

$= 6$

$\boxed{D}$

(21) H.A. $\rightarrow \lim_{x \to \infty} f(x)$

$\lim_{x \to \infty} \frac{20x^2 - x}{1 + 4x^2}$ $\quad$ deg N = deg D look @ coefficients

$= \frac{20}{4}$

$y = 5$ $\quad$ $\boxed{E}$

(22) abs max value

$\rightarrow$ rel max + end pts

$f'(x) = \frac{x(\frac{1}{x}) - \ln x(1)}{x^2}$

$0 = \frac{1 - \ln x}{x^2}$

$0 = 1 - \ln x$

$\cancel{e^{\ln x}} = e^1$

$x = e$

f': $+$ $-$; 0, e, e^2

$f(e) = \frac{\ln e}{e} = \frac{1}{e}$

$\boxed{B}$ abs max value is $\frac{1}{e}$

(23) Linear growth $\rightarrow \frac{dP}{dt}$ = constant

b/c lines have constant slopes

$\boxed{A}$ $\quad$ $\frac{dP}{dt} = 200$

(24) crit pt $\rightarrow f' = 0$

$g(x) = x^2 e^{kx}$

$g'(x) = e^{kx}(2x) + x^2(e^{kx} \cdot k)$

$0 = 2xe^{kx} + kx^2 e^{kx}$

$0 = e^{kx}(2x + kx^2)$ $\quad$ @ $x = \frac{2}{3}$

$\cancel{e^{kx}} = 0$ $\quad$ $2(\frac{2}{3}) + k(\frac{2}{3})^2 = 0$

$\frac{4}{3} + \frac{4}{9}k = 0$

$\frac{4}{9}k = -\frac{4}{3}$

$\boxed{A}$ $\quad$ $k = -3$

(25) $\frac{dy}{dx} = 2\sin x$

$\int dy = \int 2\sin x\,dx$

$y = -2\cos x + C$ $\quad$ $y(\pi) = 1$

$1 = -2\cos\pi + C$

$1 = -2(-1) + C$

$1 = 2 + C$

$-1 = C$

$\boxed{E}$ $\quad$ $y = -2\cos x - 1$

(26) $g'(x) = \int_0^x e^{-t^2}\,dt$

$g''(x) = e^{-x^2}$

$\cancel{0 = e^{-x^2}}$

g': $+$; 0, 1, 2 $\leftarrow$ b/c e^{-t^2} above x-axis, so area will be > 0

g increasing

g'': $+$; 0, 1, 2

g concave up

$\boxed{A}$

(27) $(x+2y) \cdot \frac{dy}{dx} = 2x - y$

$\frac{dy}{dx} = \frac{2x-y}{x+2y}$ $\quad$ $\left.\frac{dy}{dx}\right|_{(3,0)} = \frac{2(3)-0}{3+2(0)} = \frac{6}{3} = 2$

$\frac{d^2y}{dx^2} = \frac{(x+2y)(2 - \frac{dy}{dx}) - (2x-y)(1 + 2\frac{dy}{dx})}{(x+2y)^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(3,0)} = \frac{(3+0)(2-2) - (2 \cdot 3 - 0)(1 + 2 \cdot 2)}{(3 + 2 \cdot 0)^2}$

$= \frac{-6(5)}{9}$

$\boxed{A}$ $\quad$ $= \frac{-10}{3}$

(28) $x(t) = \sin t - \cos t$

$v(t) = x'(t) = \cos t + \sin t$

$0 = \cos t + \sin t$

$\sin t = -\cos t$

$t = 3\pi/4$

$a(t) = -\sin t + \cos t$

$a(\frac{3\pi}{4}) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}$

$= -\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}$

$= -\frac{2\sqrt{2}}{2}$

$\boxed{A}$ $\quad$ $= -\sqrt{2}$