



AP[®] Calculus AB 2010 Free-Response Questions

The College Board

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the College Board is composed of more than 5,700 schools, colleges, universities and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,800 colleges through major programs and services in college readiness, college admission, guidance, assessment, financial aid and enrollment. Among its widely recognized programs are the SAT[®], the PSAT/NMSQT[®], the Advanced Placement Program[®] (AP[®]), SpringBoard[®] and ACCUPLACER[®]. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities and concerns.

© 2010 The College Board. College Board, ACCUPLACER, Advanced Placement Program, AP, AP Central, SAT, SpringBoard and the acorn logo are registered trademarks of the College Board. Admitted Class Evaluation Service is a trademark owned by the College Board. PSAT/NMSQT is a registered trademark of the College Board and National Merit Scholarship Corporation. All other products and services may be trademarks of their respective owners. Permission to use copyrighted College Board materials may be requested online at: www.collegeboard.com/inquiry/cbpermit.html.

Visit the College Board on the Web: www.collegeboard.com.

AP Central is the official online home for the AP Program: apcentral.collegeboard.com.



CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

rate
accumulate →

rate
remove →

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

- (a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

$$\begin{aligned} \text{accumulated Snow} &= \int_0^6 7te^{\cos t} dt \\ &= 142.275 \text{ ft}^3 \end{aligned}$$

1 pt - integral

1 pt - answer

Do not write beyond this border.

Do not write beyond this border.

- (b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

↳ difference in rate of snow

t=8

$$\begin{aligned} \text{rate of volume of snow} &= \text{rate accumulated} - \text{rate removed} \\ &= f(8) - g(8) \\ &= 48.417 - 108 \\ &= -59.583 \text{ ft}^3/\text{hr} \end{aligned}$$

1 pt - answer

Continue problem 1 on page 5.

- (c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.

amount snow removed = $\int_0^x g(t) dt$

$$h(t) = \begin{cases} \int_0^0 0 dt & 0 \leq t < 6 \\ h(6) + \int_6^t 125 dx & 6 \leq t < 7 \\ h(7) + \int_7^t 108 dx & 7 < t \leq 9 \end{cases} = \begin{cases} 0 & 0 \leq t < 6 \\ 125t - 750 & 6 \leq t < 7 \\ 108t - 631 & 7 < t \leq 9 \end{cases}$$

1 pt - $h(t)$ for $0 \leq t < 6$
 1 pt - $h(t)$ for $6 \leq t < 7$
 1 pt - $h(t)$ for $7 < t \leq 9$

$$\begin{aligned} h(6) + \int_6^t 125 dx &= h(7) + \int_7^t 108 dx \\ = 0 + (125x) \Big|_6^t &= 125 + (108x) \Big|_7^t \\ = 0 + 125t - 125(6) &= 125 + 108t - 108(7) \\ = 125t - 750 &= 108t - 631 \end{aligned}$$

- (d) How many cubic feet of snow are on the driveway at 9 A.M.?

$\int \text{rate accum} - \int \text{rate remove}$

$$\begin{aligned} \text{snow on driveway} &= \int_0^9 f(t) dt - \int_0^9 g(t) dt \\ &= \int_0^9 f(t) dt - \left[\int_0^6 g(t) dt + \int_6^7 g(t) dt + \int_7^9 g(t) dt \right] \\ &= 367.335 - [0 + 125 + 216] \\ &= 26.335 \text{ ft}^3 \end{aligned}$$

1 pt - $\int_0^9 f(t) dt$
 1 pt - $\int_0^9 g(t) dt$
 or $h(9)$

1 pt - answer

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

| | | | | | |
|---------------------------------|---|---|----|----|----|
| t (hours) | 0 | 2 | 5 | 7 | 8 |
| $E(t)$ (hundreds of entries) | 0 | 4 | 13 | 21 | 23 |

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \leq t \leq 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

- (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

$$E'(6) = \frac{E(7) - E(5)}{7 - 5} \quad \begin{array}{l} \text{hundreds of entries} \\ \text{hrs} \end{array}$$

$$= \frac{21 - 13}{2}$$

$$= 4 \text{ hundreds of entries/hr}$$

1 pt - answer

- (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8} \int_0^8 E(t) dt$.

Using correct units, explain the meaning of $\frac{1}{8} \int_0^8 E(t) dt$ in terms of the number of entries.

$$\frac{1}{8} \int_0^8 E(t) dt = \frac{1}{8} \left[\frac{1}{2}(0+4)(2) + \frac{1}{2}(4+13)(3) + \frac{1}{2}(13+21)(2) + \frac{1}{2}(21+23)(1) \right]$$

$$= 10.688 \text{ hundreds of entries}$$

avg value!

1 pt - trap sum
1 pt - approx

$\frac{1}{8} \int_0^8 E(t) dt$ means avg # of entries, in hundreds, deposited from $t=0$ to $t=8$ hrs (noon to 8pm)

1 pt - meaning

Do not write beyond this border.

$\frac{1}{\text{hr}} \left(\frac{1}{2} \text{ (entries) (hr)} \right)$

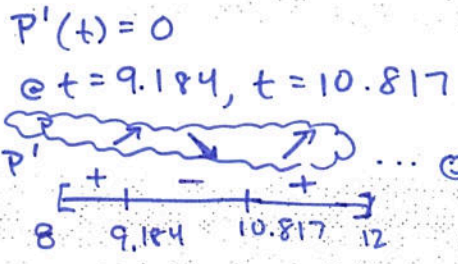
(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P , where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)? ↳ processed time period

rate processed →

$$\begin{aligned}
 \# \text{ entries not processed} &= \# \text{ deposited @ } t=12 - \# \text{ processed @ } t=12 \\
 &= E(12) - \int_8^{12} P(t) dt \\
 &= 23 - 16 \\
 &= 7 \text{ hundred entries}
 \end{aligned}$$

↳ pt-integral
↳ pt-answer

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.



↳ process max
 $P'(t) = 0$
 $P'(t)$ change pos to neg
 + check endpts
 ↳ pt - set
 $P'(t) = 0$
 ↳ pt - crit #s

↳ rel. max @ $t = 9.184$ b/c
 P' changes from pos to neg
 @ $t = 9.184$

endpt → $P(8) = 0$
 $P(9.184) = 5.089$
 endpt → $P(12) = 8$

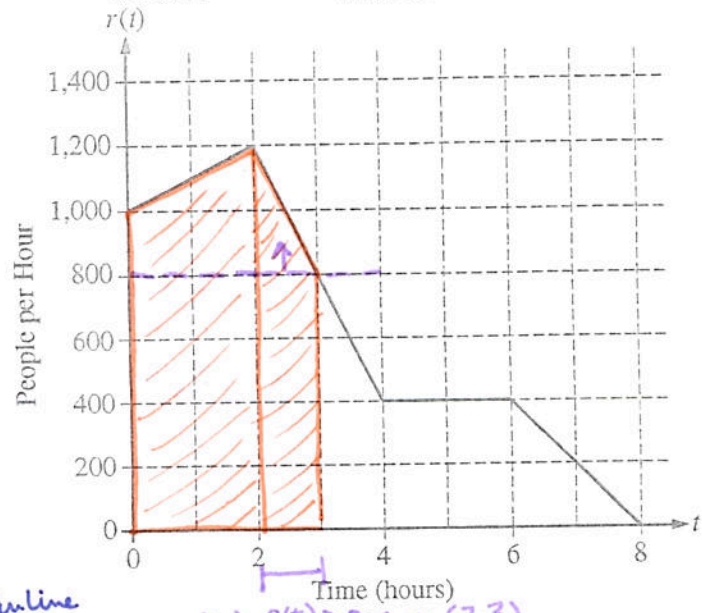
↳ pt - answer w/ reason

Entries processed most quickly @ $t = 12$

Do not write beyond this border.

Do not write beyond this border.

graph of rate people arrive →



initial value wait in line

3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

Do not write beyond this border.

people arrive = $\int_0^3 r(t) dt$ (area) \rightarrow rate arrive

$$= \frac{1}{2} (1000 + 1200)(2) + \frac{1}{2} (1200 + 800)(1)$$

$$= 3200 \text{ people}$$

1 pt - integral

1 pt - answer

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

\rightarrow rate waiting > 0 or < 0

$$\text{rate waiting} = \text{rate arrive} - \text{rate move on ride}$$

$$W'(t) = r(t) - 800$$

btw (2,3), $r(t) - 800 > 0$
 b/c $r(t) > 800$

1 pt - answer w/ reason

Since $r(t) > 800$ btw $t = 2$ + $t = 3$, # people waiting in line is increasing

Continue problem 3 on page 9.

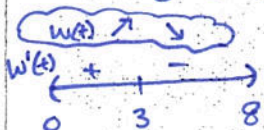
- (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

$$W'(t) = 0$$

$$r(t) - 800 = 0$$

$$r(t) = 800$$

$$t = 3$$



\hookrightarrow line longest @ $t=3$ b/c W' changes from pos to neg @ $t=3$

(W' only changes signs @ $t=3$ so no need to check endpoints)

$$\begin{aligned} W(3) &= 700 + \int_0^3 (r(t) - 800) dt = 700 + 3200 - (800t) \Big|_0^3 \\ &= 700 + \int_0^3 r(t) dt - \int_0^3 800 dt = 700 + 3200 - 800(3) \\ &= 1500 \text{ people} \end{aligned}$$

1 pt - identify $t=3$

1 pt - reason

1 pt - # people in line

- (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

$$\begin{aligned} \hookrightarrow \text{wait} &= 0 \\ W(t) &= 0 \end{aligned}$$

$$W(t) = 700 + \int_0^t (r(x) - 800) dx$$

$$0 = 700 + \int_0^t (r(x) - 800) dx$$

1 pt - $800t$
1 pt - integral
1 pt - answer

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.