CollegeBoard
Advanced Placement Program

AP® Calculus AB
2010 Free-Response Questions

The College Board

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

rate accumulate

rate remove

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

accumulated Snow $= \int_0^6 7te^{\cos t}\,dt$ — 1pt - integral

$= 142.275 \text{ ft}^3$ — 1pt - answer

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(b) Find the rate of change of the volume of snow on the driveway at $\boxed{8 \text{ A.M.}}$ $t=8$

difference in rate of snow

rate of volume of snow = rate accumulated − rate removed

$= f(8) - g(8)$

$= 48.417 - 108$

$= -59.583 \text{ ft}^3/\text{hr}$ — 1pt - answer

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Continue problem 1 on page 5.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

amount snow removed $= \int_0^x g(t)\,dt$

$$h(t) = \begin{cases} \int_0^0 0\,dt & 0 \le t < 6 \\ h(6) + \int_6^t 125\,dx & 6 \le t < 7 \\ h(7) + \int_7^t 108\,dx & 7 < t \le 9 \end{cases} = \begin{cases} 0 & 0 \le t < 6 \\ 125t - 750 & 6 \le t < 7 \\ 108t - 631 & 7 < t \le 9 \end{cases}$$

1pt - h(t) for $0 \le t < 6$
1pt - h(t) for $6 < t \le 7$
1pt - h(t) for $7 < t \le 9$

$h(6) + \int_6^t 125\,dx$
$= 0 + (125x)\Big|_6^t$
$= 0 + 125t - 125(6)$
$= 125t - 750$

$h(7) + \int_7^t 108\,dx$
$= 125 + (108x)\Big|_7^t$
$= 125 + 108t - 108(7)$
$= 108t - 631$

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

$\int$ rate occur $- \int$ rate remove

snow on driveway $= \int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$

$= \int_0^9 f(t)\,dt - \left[\int_0^6 g(t)\,dt + \int_6^7 g(t)\,dt + \int_7^9 g(t)\,dt\right]$

$= 367.335 - [0 + 125 + 216]$

$= 26.335\ ft^3$

1pt - $\int_0^9 f(t)\,dt$
1pt - $\int_0^9 g(t)\,dt$ or $h(9)$

1pt - answer

GO ON TO THE NEXT PAGE.

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t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

↳ deposited

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

$$E'(6) = \frac{E(7) - E(5)}{7-5}$$

hundreds of entries / hrs

$$= \frac{21-13}{2}$$

$$= 4 \text{ hundreds of } \frac{\text{entries}}{\text{hr}}$$

1pt - answer

avg value!

(b) Use a trapezoidal sum with the $\boxed{\text{four}}$ subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

$$\frac{1}{8}\int_0^8 E(t)\,dt = \frac{1}{8}\left[\frac{1}{2}(0+4)(2) + \frac{1}{2}(4+13)(3) + \frac{1}{2}(13+21)(2) + \frac{1}{2}(21+23)(1)\right]$$

1pt - trap sum

$\frac{1}{\cancel{hr}}\left(\frac{1}{2}(\text{hundred entries})(\cancel{hr})\right)$

$$= 10.688 \text{ hundreds of entries}$$

1pt - approx

$\frac{1}{8}\int_0^8 E(t)\,dt$ means avg # of entries, in hundreds, deposited from $t=0$ to $t=8$ hrs (noon to 8pm)

1pt - meaning

Continue problem 2 on page 7.

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(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

rate processed

$\underbrace{8 \le t \le 12}_{\text{processed time period}}$

$$\#\text{ entries not processed} = \underset{@t=12}{\#\text{ deposited}} - \underset{@t=12}{\#\text{ processed}}$$

$$= E(12) - \int_8^{12} P(t)\,dt$$

1pt - integral

$$= 23 - 16$$

$$= 7 \text{ hundred entries}$$

1pt - answer

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

process max

$P'(t) = 0$

$P'(t)$ change pos to neg + check endpts

$P'(t) = 0$

$@\, t = 9.184,\ t = 10.817$

P ↗ ↘ ↗ ... ☺

P': + on $[8, 9.184]$, − on $[9.184, 10.817]$, + on $[10.817, 12]$

1pt - set $P'(t)=0$

1pt - crit #s

rel. max @ $t = 9.184$ b/c P' changes from pos to neg @ $t = 9.184$

endpt → $P(8) = 0$

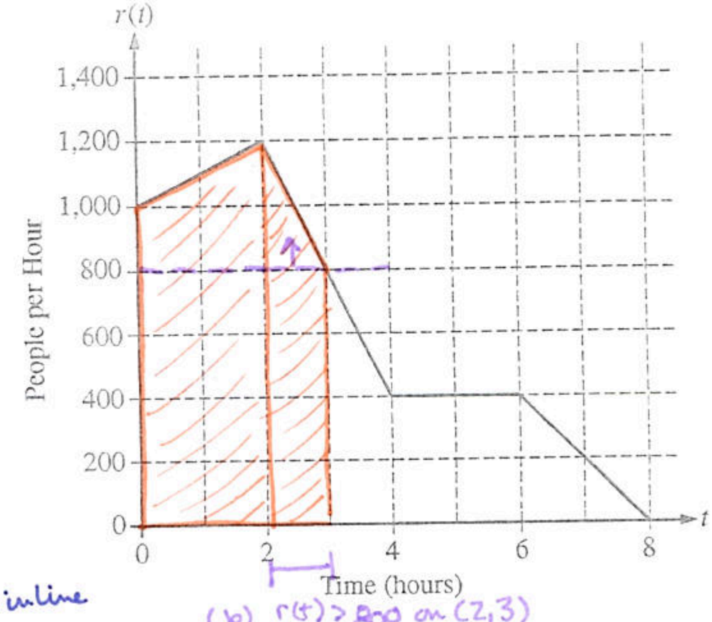
$P(9.184) = 5.089$

endpt → $P(12) = 8$

1pt - answer w/ reason

Entries processed most quickly @ $t = 12$

graph → of rate people arrive



3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) How many people arrive at the ride between $\boxed{t = 0 \text{ and } t = 3}$? Show the computations that lead to your answer. ↳ $\int$ rate arrive

people arrive $= \int_0^3 r(t)\,dt$ (area) :)

1pt – integral

$= \frac{1}{2}(1000+1200)(2) + \frac{1}{2}(1200+800)(1)$

1pt – answer

$= 3200$ people

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer. ↳ rate waiting > 0 or < 0

rate waiting = rate arrive − rate move on ride

$W'(t) = r(t) - 800$

btn $(2,3)$, $r(t) - 800 > 0$

b/c $r(t) > 800$

1 pt – answer w/ reason

Since $r(t) > 800$ btn $t=2$ + $t=3$, # people waiting in line is increasing

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Continue problem 3 on page 9.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

↳ wait max

$w'(t) = 0$

w' change pos to neg

+ ~~check endpts~~

$w'(t) = 0$

$r(t) - 800 = 0$

$r(t) = 800$

$t = 3$

$w(t)$ ↗ ↘

$w'(t)$ + −

0 3 8

1pt - identify $t = 3$

1pt - reason

↳ Line Longest @ $t = 3$ b/c w' changes from pos to neg @ $t = 3$

(w' only changes signs @ $t = 3$ so no need to check endpts)

$$w(3) = 700 + \int_0^3 (r(t) - 800)\,dt$$

$$= 700 + \int_0^3 r(t)\,dt - \int_0^3 800\,dt$$

$$= 700 + 3200 - (800t)\Big|_0^3$$

$$= 700 + 3200 - 800(3)$$

$$= 1500 \text{ people}$$

1pt - # people in line

(d) Write, but $\boxed{\text{do not solve}}$, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

↳ wait = 0

$w(t) = 0$

$$w(t) = 700 + \int_0^t (r(x) - 800)\,dx$$

$$0 = 700 + \int_0^t (r(x) - 800)\,dx$$

1pt - 800t

1pt - integral

1pt - answer

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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