NO CALCULATOR ALLOWED

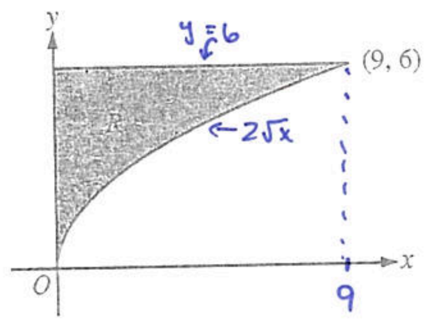
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R. → top - bottom ... ☺

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$$\text{Area of } R = \int_0^9 (6 - 2\sqrt{x})\,dx$$

1pt - integrand

$$= \int_0^9 (6 - 2x^{1/2})\,dx$$

$$= 6x - 2\left(\tfrac{2}{3}x^{3/2}\right)\Big|_0^9$$

1pt - antiderivative

$$= 6(9) - \tfrac{4}{3}(9)^{3/2} - (0)$$

1pt - answer
ok to stop here

$$= 54 - \tfrac{4}{3}(\sqrt{9})^3$$

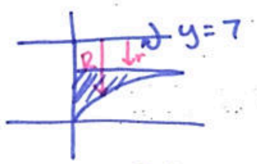
$$= 54 - \tfrac{4}{3}\cdot 27$$

$$= 54 - 36$$

$$= 18$$

Continue problem 4 on page 11.

(b) Write, but $\boxed{\text{do not evaluate}}$, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.



* WASHER *

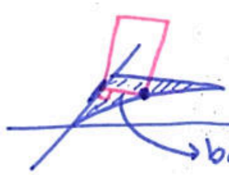
outside radius — inside radius

$$V = \pi \int_0^9 \left[(7 - 2\sqrt{x})^2 - (7-6)^2 \right] dx$$

2pts - integrand
1pt - limits + constant (π)

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a $\boxed{\text{rectangle}}$ whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$A = bh$ $\quad$ $h = 3\,\text{base}$



$y = 2\sqrt{x}$

$\frac{y}{2} = \sqrt{x} \rightarrow x = \left(\frac{y}{2}\right)^2$

base $= \frac{y^2}{4} - 0$

$A = bh$

$A = b(3b)$

$A = \left(\frac{y^2}{4}\right)\left(3 \cdot \frac{y^2}{4}\right)$

$A = 3\left(\frac{y^2}{4}\right)^2$

y #s

$$V = \int_0^6 \left(3\left(\frac{y^2}{4}\right)^2 \right) dy$$

2pt - integrand
1pt - answer

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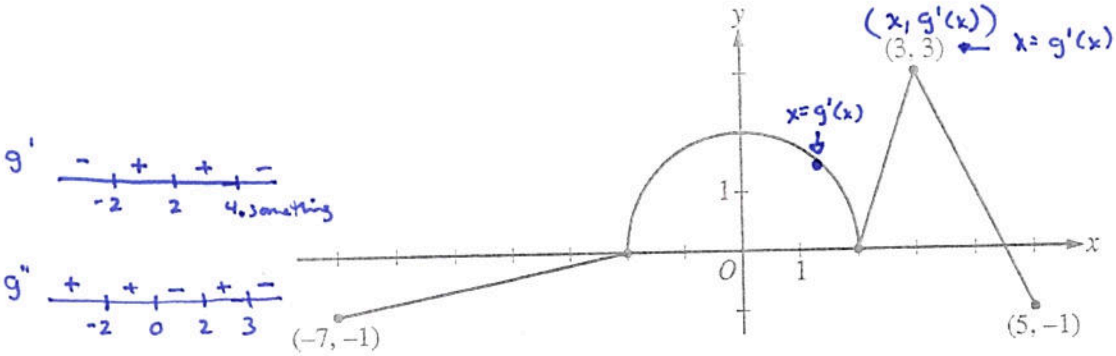

Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

$$g(3) = 5 + \int_0^3 g'(t)\,dt$$
$$= 5 + \tfrac{1}{4}(\pi(2)^2) + \tfrac{1}{2}(1)(3)$$
$$= 6.5 + \pi$$

1pt - uses initial condition

1pt - g(3)

$$g(-2) = 5 + \int_0^{-2} g'(t)\,dt$$
$$= 5 + -\tfrac{1}{4}\pi(2)^2$$
$$= 5 - \pi$$

1pt - g(-2)

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Continue problem 5 on page 13.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

↳ g" changes signs

g has inf pts @ $x = 0$, $x = 2$, and $x = 3$

b/c g" changes signs @ $x = 0, x = 2, x = 3$

1pt – $x=0$, $x=2$, $x=3$

1pt – reason

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

$h'(x) = 0$ or DNE

$h'(x) = g'(x) - x$

$0 = g'(x) - x$

$x = g'(x)$

1pt – $h'(x)$

@ $x = 3$

1pt – crit #s, $\sqrt{2}$, and 3

equation of $g'(x)$ on $(-2, 2)$ is semicircle

$x^2 + y^2 = 2^2$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

so $\sqrt{4 - x^2} = x$

1pt – rel max @ $\sqrt{2}$ w/ reason

$4 - x^2 = x^2$

$4 = 2x^2$

$2 = x^2$

1pt – neither @ $x=3$ w/ reason

$\pm\sqrt{2} = x$

h' + | − | −
0 $\sqrt{2}$ 2 3 4

h has rel. max @ $x = \sqrt{2}$ b/c h' changes from pos to neg @ $x = \sqrt{2}$

h has neither max nor min @ $x = 3$ b/c h' doesn't change signs @ $x = 3$

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6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

$y - y_1 = m(x - x_1)$

$\left.\frac{dy}{dx}\right|_{(1,2)} = 1(2)^3 = 8$

1pt - $\left.\frac{d^2y}{dx^2}\right|_{(1,2)}$

$y - 2 = 8(x - 1)$

1pt - answer

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(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

need to look @ concavity

concup line under

condown line over

$y - 2 = 8(x - 1)$

$y - 2 = 8(1.1 - 1)$

$y - 2 = 8(.1)$

$y = 2.8$

$f(1.1) = 2.8$

1pt - approx of $f(1.1)$

$\frac{d^2y}{dx^2}$ on $(1, 1.1)$ interval $= y^3(1 + 3x^2y^2)$

$f(x) > 0$ so $y > 0$

$= (+\#)^3(1 + 3x^2(+\#)^2)$

> 0

1pt - less than w/ reason

approx for $f(1.1)$ is less than $f(1.1)$ b/c $\frac{d^2y}{dx^2} > 0$ on $(1, 1.1)$

Continue problem 6 on page 15.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

y's w/ dy's and x's w/ dx's

$\frac{dy}{dx} = xy^3$

$dy = xy^3\,dx$

$\frac{1}{y^3}\,dy = x\,dx$ 1pt-separate vars.

$\int y^{-3}\,dy = \int x\,dx$

$-\frac{1}{2}y^{-2} = \frac{1}{2}x^2 + C$ 1pt - antiderive 1pt - "+C"

$-\frac{1}{2}(2)^{-2} = \frac{1}{2}(1)^2 + C$ 1pt - initial condition

$-\frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{2} + C$

$-\frac{1}{8} - \frac{1}{2}$

$-\frac{1}{8} - \frac{4}{8}$

$-\frac{5}{8} = C$

$-\frac{1}{2}y^{-2} = \frac{1}{2}x^2 - \frac{5}{8}$

$y^{-2} = -x^2 + \frac{5}{4}$

$\frac{1}{y^2} = \frac{-4x^2+5}{4}$

$y^2 = \frac{4}{-4x^2+5}$

$y = \pm\sqrt{\frac{4}{-4x^2+5}}$

$y>0$, so, $y = \sqrt{\frac{4}{-4x^2+5}}$ 1pt - solve for y

or $\frac{2}{\sqrt{-4x^2+5}}$

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