CollegeBoard
Advanced Placement Program

AP® Calculus AB
2010 Free-Response Questions
Form B

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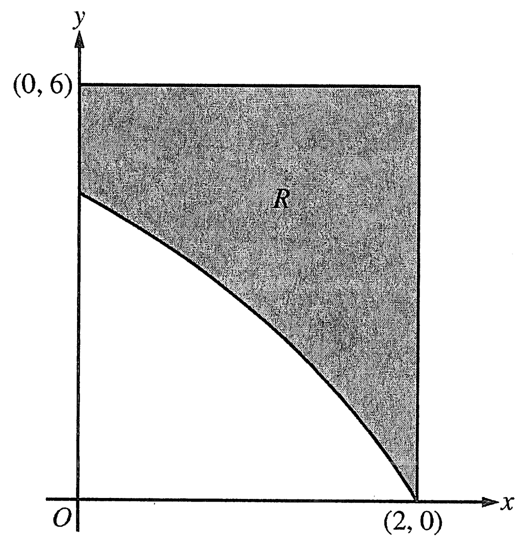
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

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Continue problem 1 on page 5.

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(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

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2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

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Continue problem 2 on page 7.

(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

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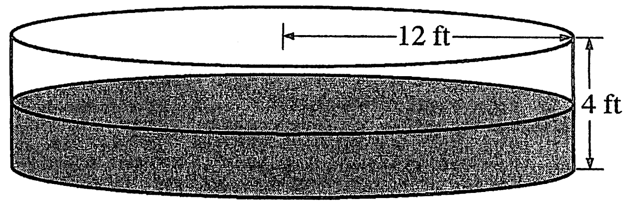
t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

Continue problem 3 on page 9.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

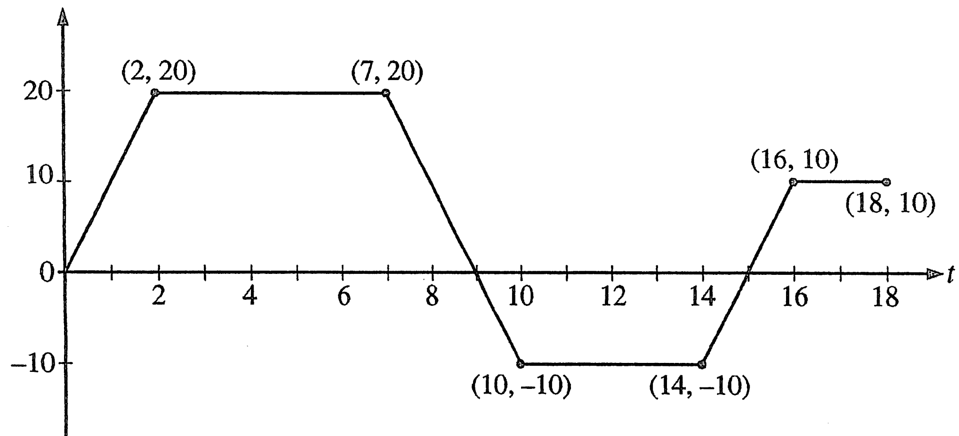
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A ? How far from building A is the squirrel at that time?

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Continue problem 4 on page 11.

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

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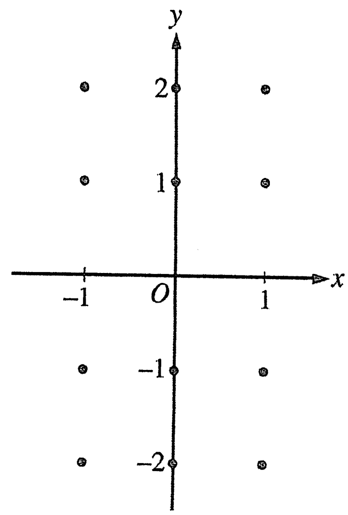
NO CALCULATOR ALLOWED

5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

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Continue problem 5 on page 13.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

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6. Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

Continue problem 6 on page 15.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

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