NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

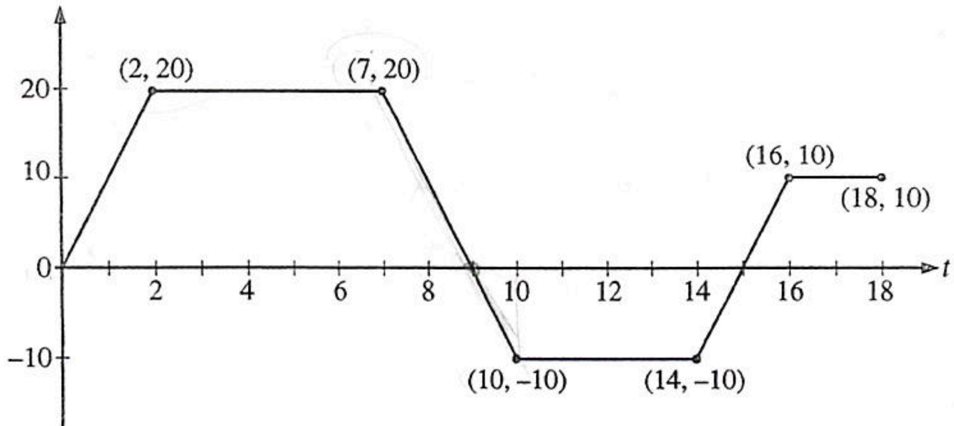
Number of problems—3

No calculator is allowed for these problems.

x ↗ ↘ ↗

v + − +

0 9 15 18



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

↳ velocity change signs

@ $t = 9$ and $t = 15$, squirrel changes direction b/c $v(t)$ changes signs.

1pt - t-value

1pt - reason

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?

↳ max distance → $\int_0^t v(x)\,dx$

$X(0) = 0$

$X(9) = \int_0^9 v(t)\,dt = \frac{1}{2}(9+5)(20) = 140$

$X(15) = 140 + \int_9^{15} v(t)\,dt = 140 + \frac{1}{2}(6+4)(-10) = 90$

$X(18) = 90 + \int_{15}^{18} v(t)\,dt = 90 + \frac{1}{2}(3+2)(10) = 115$

Squirrel farthest @ $t = 9$. Squirrel is 140 units from building A

1pt - crit + endpts

1pt - answers

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Continue problem 4 on page 11.

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

$\hookrightarrow \int |v(t)|\, dt$

total distance $= \int_0^{18} |v(t)|\, dt$

$= 140 + 50 + 25$

$= 215$

1pt – answer

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

$\hookrightarrow$ line $y - y_1 = m(x - x_1)$

$y - 20 = -10(x - 7)$

$y - 20 = -10x + 70$

$y = -10x + 90$

$a(t) = v'(t) =$

$= \dfrac{20 - (-10)}{7 - 10}$

$a(t) = -10$

$v(t) = -10t + 90$

$x(t) = \int (-10t + 90)\, dt$

$x(t) = -5t^2 + 90t + C \rightarrow x(t) = -5t^2 + 90t - 265$

$x(9) = -5(9)^2 + 90(9) + C$

$140 = -405 + 810 + C$

$-265 = C$

1pt – $a(t)$

1pt – $v(t)$

1pt – $x(t)$

$81 \times 5 = 405$

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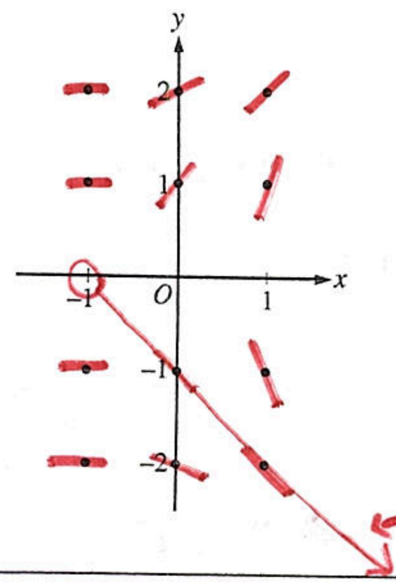
5. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

$y \backslash x$	-1	0	1
2	0	$\frac{1}{2}$	1
1	0	1	2
-1	0	-1	-2
-2	0	$-\frac{1}{2}$	-1



← solution curve thru $(0,-1)$

1pt - zero slopes

1pt - nonzero slopes

1pt - curve through $(0,-1)$

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\dfrac{dy}{dx} = -1$.

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$-1 = \frac{x+1}{y}$$

$$-y = x+1$$

$$y = -x-1$$

1pt - description

all pts where $\frac{dy}{dx} = -1$ when $y = -x-1$ and $y \neq 0$

Continue problem 5 on page 13.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$\int y\,dy = \int (x+1)\,dx$$

1pt - separate vars

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C$$

1pt - antiderive

1pt - "+C"

$$\frac{1}{2}(-2)^2 = C$$

1pt - initial condition

$$2 = C$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + 2$$

$$y^2 = x^2 + 2x + 4$$

$$y = \pm\sqrt{x^2+2x+4}$$

1pt - solve for y

$$y = -\sqrt{x^2+2x+4}$$

b/c y<0 ... ☺

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6. Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

↳ positive velocity

$r'(t) = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = 3(t-3)(t-1)$

$t = 1, t = 3$

1pt – $r'(t)$

r': [+ | − | +]
0 .5 1 2 3 4 6

1pt – answer

R moving right on $(0,1) \cup (3,6)$ b/c $r'(t) > 0$ on those intervals.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

↳ $r'(t)$ + $p'(t)$ opposite signs

1pt – $p'(t)$

$p'(t) = -2\sin\left(\frac{\pi}{4}t\right) \cdot \frac{\pi}{4}$

$0 = -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right)$

$0 = \sin\left(\frac{\pi}{4}t\right)$

$\sin^{-1}(0) = \frac{\pi}{4}t$

$0 = \frac{\pi}{4}t \qquad \pi = \frac{\pi}{4}t$

$0 = t \qquad 4 = t$

Two particles travel in opposite directions on $(0,1) \cup (3,4)$ b/c $r'(t)$ + $p'(t)$ have opposite signs on those intervals

p': (− | +)
0 1 4 5 6

$p' < 0$ on $(1,4)$

$p' > 0$ on $(4,6)$

1pt – answer

1pt – describe $p' > 0$ + < 0

Continue problem 6 on page 15.

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$\to$ a(t) & v(t) different signs

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

$\hookrightarrow$ a(t) & v(t) same signs

$$p''(t) = a(t)$$
$$= -\frac{\pi}{2}\cos\left(\frac{\pi}{4}t\right)\cdot\frac{\pi}{4}$$
$$a(3) = -\frac{\pi}{2}\cos\left(\frac{\pi}{4}\cdot 3\right)\cdot\frac{\pi}{4}$$
$$= -\frac{\pi}{2}\cdot -\frac{\sqrt{2}}{2}\cdot\frac{\pi}{4}$$
$$= \frac{\pi^2\sqrt{2}}{16}$$

$p'(3) < 0$ (from # line)

1pt - $p''(3)$

1pt - answer w/ reason

Since $a(3) > 0$ and $p'(3) < 0$, particle P is slowing down @ $t = 3$.

(d) Write, but do not evaluate, an expression for the average $\boxed{\text{distance between}}$ the two particles on the interval $1 \le t \le 3$.

$\hookrightarrow$ distance btn. $=$ | diff. in position |

~~$\frac{1}{3-1}\int_1^3 |\text{distance}|\,dt$~~

$$\frac{1}{3-1}\int_1^3 \Big|\underbrace{p(t) - r(t)}_{\text{distance btn 2 particles}}\Big|\,dt$$

1pt - integrand

$$\frac{1}{2}\int_1^3 |p(t) - r(t)|\,dt$$

1pt - limits & constant

GO ON TO THE NEXT PAGE.

$$g(.3) = 2 + \int_{0}^{.3} g'(x)\,dx$$

$$= g(1) + g(x) \Big|_{0}^{.3}$$

$$= g(1) + g(.3) - g(0)$$

$$g(x) = g(1) + \int_{1}^{x} g'(t)\,dt$$