CollegeBoard
Advanced Placement
Program

AP® Calculus AB
2011 Free-Response Questions

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

↳ $v(t)$ & $a(t)$ same signs ↳ $v(t)$ & $a(t)$ different signs

$v(5.5) = -.453$

$a(5.5) = -1.359$

Speed inc @ $t = 5.5$ b/c $v(5.5) < 0$ and $a(5.5) < 0$

2 pts - conclusion w/ reason

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

avg velocity $= \frac{1}{6-0}\int_0^6 v(t)\,dt$

$= 1.949$

1 pt - integral

1 pt - answer

avg velocity $= \frac{x(b)-x(a)}{b-a}$ same

or

avg velocity $= \frac{1}{b-a}\int_a^b v(t)\,dt$

$= \frac{1}{b-a}\,x(t)\Big|_a^b$

$= \frac{1}{b-a}(x(b)-x(a))$

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Continue problem 1 on page 5.

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(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

↳ $\int |v(t)|\,dt$

total distance $= \int_0^6 |v(t)|\,dt$

$= 12.573$

1 pt - integral

1 pt - answer

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

↳ $v(t)$ changes signs ↳ $x(t)$ = initial pos.

@ $t = 5.196$, particle changes direction b/c $v(t)$ changes signs @ $t = 5.196$

1 pt - considers $v(t) = 0$

$x(5.196) = x(0) + \int_0^{5.196} v(t)\,dt$

$= 2 + \int_0^{5.196} v(t)\,db$

$= 14.135$

1 pt - integral

1 pt - answer

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t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2 3 4 1

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

$H'(3.5) = \dfrac{H(5) - H(2)}{5 - 2}$

$= \dfrac{52 - 60}{5 - 2}$ ← ok to stop here

$= -\frac{8}{3}$ °C/min

1 pt - answer

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

avg value

$\frac{1}{2}(b_1 + b_2)h$

$\frac{1}{10}\int_0^{10} H(t)\,dt = \frac{1}{10}\left[\frac{1}{2}(66+60)(2) + \frac{1}{2}(60+52)(3) + \frac{1}{2}(52+44)(4) + \frac{1}{2}(44+43)(1)\right]$

← ok to stop here

$= 52.95°\text{C}$

1 pt - trap sum

1 pt - estimate

$\frac{1}{10}\int_0^{10} H(t)\,dt$ means average temp of the pot of tea in °C from $t = 0$ to $t = 10$ minutes

1 pt - meaning w/ units

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Continue problem 2 on page 7.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

$$\int_0^{10} H'(t)\,dt = H(t)\Big|_0^{10}$$

$$= H(10) - H(0)$$

$$= 43 - 66 \quad \leftarrow \text{ok to stop here}$$

$$= -23^\circ C$$

1 pt - value of integral

$\int_0^{10} H'(t)\,dt$ means the change (difference) in temp of pot of tea in °C from $t = 0$ to $t = 10$ minutes

1 pt - meaning w/ units

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

temp biscuits @ $t = 10$ → $B(10) = 100^\circ + \int_0^{10} B'(t)\,dt$

$$= 34.183^\circ C$$

1 pt - integrand

1 pt - initial condition

temp of tea → $H(10) = 43^\circ C$ (from table)

The biscuits are $8.817^\circ C$ cooler than the tea @ $t = 10$

1 pt - answer

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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