



AP[®] Calculus AB
2011 Free-Response Questions
Form B

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CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S , where $S(t)$ is measured in millimeters and t is measured in days for $0 \leq t \leq 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2 \sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

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(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

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- (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M , where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \leq t \leq 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \leq t \leq 60$ to justify that there exists a time t , $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

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2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

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(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

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(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

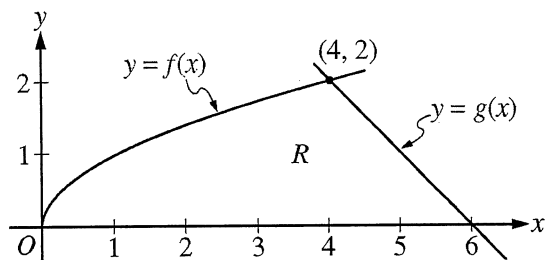
CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.
- (a) Find the area of R .

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NO CALCULATOR ALLOWED

- (b) The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

- (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .

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NO CALCULATOR ALLOWED

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

- (a) Find the x -coordinate of the critical point of f . Determine whether the point is a relative maximum, a relative minimum, or neither for the function f . Justify your answer.

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- (b) Find all intervals on which the graph of f is concave down. Justify your answer.

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NO CALCULATOR ALLOWED

(c) Given that $f(1) = 2$, determine the function f .

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NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t , measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t .

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

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(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)| dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)| dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

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NO CALCULATOR ALLOWED

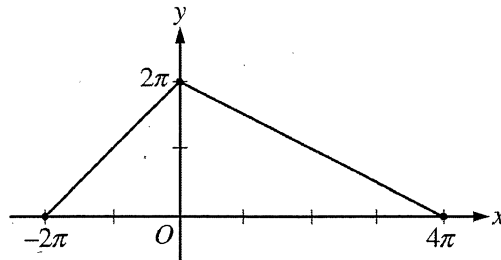
(c) For $40 \leq t \leq 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t , the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

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NO CALCULATOR ALLOWED

Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and

$$\text{let } f(x) = g(x) - \cos\left(\frac{x}{2}\right).$$

(a) Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.

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(b) Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t) dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

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