CollegeBoard
Advanced Placement Program

AP® Calculus AB
2011 Free-Response Questions
Form B

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CALCULUS AB
SECTION II, Part A
Time—30 minutes

Number of problems—2

$V = \pi r^2 h$

A graphing calculator is required for these problems.

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

$S(60) = 0 + \int_0^{60} S'(t)\,dt$

$= 171.813$ mm

1pt - limits

1pt - integrand

1pt - answer

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

avg rate of change $= \dfrac{S(60) - S(0)}{60 - 0}$ $\dfrac{\text{mm}}{\text{day}}$ ...

$= 2.864$ mm/day

1pt - answer

OR $\dfrac{1}{60-0}\int_0^{60} S'(t)\,dt$

$= 2.864$ mm/day

1pt - units in (b) or (c)

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Continue problem 1 on page 5.

(c) Assuming no evaporation occurs, at what $\boxed{\text{rate is the volume}}$ of water in the can changing at time $t = 7$? Indicate units of measure.

$V = \pi r^2 h$ (radius is constant) $r = 10\text{mm}$

$\frac{\text{mm}^3}{\text{day}}$ $\rightarrow$ $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ $\qquad \frac{dh}{dt} = S'(t)$ $\qquad$ 1pt – $\frac{dV}{dt}$ and $\frac{dh}{dt}$

$V'(7) = \pi(10)^2 \cdot S'(7)$

$= 602.218 \ \text{mm}^3/\text{day}$ $\qquad$ 1pt – answer

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem (cont.) to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

$\rightarrow M'(t) = S'(t)$
$M'(t) - S'(t) = 0$
$D(t) = 0$

$D(0) = M'(0) - S'(0)$
$= .825 - 1.5$
$= -.675$

$D(60) = M'(60) - S'(60)$
$= 72.825 - 3.448$
$= 69.377$

1pt – $D(0)$ and $D(60)$

$D(t)$ is cont.

Since $D(0) < 0$ and $D(60) > 0$, by IVT there is some t on $(0, 60)$ s.t. $D(t) = 0$ $\qquad$ 1pt – reason

($M'(t) - S'(t) = 0 \rightarrow M'(t) = S'(t)$, so the heights are changing at same rate)

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2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

$\lim\limits_{t \to 5^-} \dfrac{600t}{t+3} = 375$

$\lim\limits_{x \to 5^+} 1000e^{-.2t} = 367.879$

2pts - answer w/ limit work

Since $\lim\limits_{x \to 5^-} r(t) \neq \lim\limits_{x \to 5^+} r(t)$, $r(t)$ is not continuous @ $t = 5$.

(b) Find the average value rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

avg rate $= \dfrac{1}{8-0} \displaystyle\int_0^8 r(t)\,dt$

$= \dfrac{1}{8}\left[\displaystyle\int_0^5 \dfrac{600t}{t+3}\,dt + \int_5^8 1000e^{-.2t}\,dt \right]$

$= 258.053$ Liters/hr

1pt - integrand
1pt - limits & constant
1pt - answer

or

avg rate $= \dfrac{\int_0^8 r(t)\,dt - \int_0 r(t)\,dt}{8-0}$

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Continue problem 2 on page 7.

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(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

$r'(3) = 50$ liter/hr^2

r' pos @ t=3

r inc @ t=3 ...

$r(t) \rightarrow$ liter/hr

$r'(t) \rightarrow$ liter/hr^2

$\frac{\text{liter/hr}}{\text{hr}} \cdot \frac{1}{\text{hr}}$

1pt - $r'(3)$

Rate that water drains out of tank @ t = 3 hour is increasing @ 50 liters/hrs^2

1pt - meaning of $r'(3)$

(d) Write, but do not solve, an equation involving an integral to find the time $\boxed{A}$ when the amount of water in the tank is 9000 liters.

initial amount - drain

Amount of water in tank $= 12000 - \int_0^A r(t)\,dt$

1pt - integral

1pt - equation

$$9000 = 12000 - \int_0^A r(t)\,dt$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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