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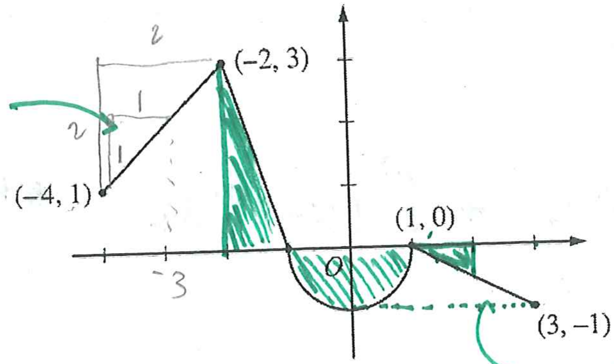
eq.
$y-1 = 1(x+4)$
$y = x+5$

$y(-3) = 2$



Graph of f

eq. $y - 0 = \frac{-1}{2}(x-1)$
$y = -\frac{1}{2}x + \frac{1}{2}$

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

$y(2) = -\frac{1}{2}(2) + \frac{1}{2}$
$= -\frac{1}{2}$

$g(2) = \int_1^2 f(t)\,dt$
$= \frac{1}{2}(1)(-\frac{1}{2})$
$= -\frac{1}{4}$

1pt - g(2)

$g(-2) = \int_1^{-2} f(t)\,dt$
$= -\int_{-2}^{1} f(t)\,dt = -\left[\left(-\frac{\pi(1)^2}{2}\right) + \frac{1}{2}(1)(3)\right] = \frac{\pi}{2} - \frac{3}{2}$

1pt - g(-2)

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$g'(x) = \frac{d}{dx}\int_1^x f(t)\,dt$
$= f(x)$

$g'(-3) = f(-3)$
$= 2$

$g''(x) = f'(x)$
$g''(-3) = f'(-3)$
$= 1$

1pt - g'(-3)
1pt - g''(-3)

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Continue problem 3 on page 15.

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$g' = 0$

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$g'(x) = 0 \to g$ has hor. tan. line

1pt – $g'(x) = 0$

$g'(x) = f(x) = 0$

@ $x = -1$, $x = 1$

g': $+$ | $-$ | $-$ (at -1, 1)

1pt – $x = -1$ and $x = 1$

g has rel. max @ $x = -1$ b/c g' changes from pos to neg.

@ $x = 1$
g has neither rel. max nor min b/c g' doesn't change signs.

1pt – answers w/ reasons

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

↳ g'' changes signs

$g''(x) = f'(x)$

$g'' = 0$ @ $x = 0$ $\quad$ g'' DNE @ $x = -2, -1, 1$

g'': $+$ | $-$ | $-$ | $+$ | $-$ (at -2, -1, 0, 1)

g has pt. of inf @ $x = -2$, $x = 1$, and $x = 0$ b/c g'' changes signs.

1pt – answer

1pt – reason

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4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

$$f'(x) = \tfrac{1}{2}(25 - x^2)^{-1/2}(-2x)$$
$$= -x(25 - x^2)^{-1/2}$$

2pts - $f'(x)$

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \tfrac{3}{4}(x + 3)$$

$$f(-3) = \sqrt{25 - (-3)^2}$$
$$= \sqrt{16}$$
$$= 4$$

$$f'(-3) = -(-3)(25 - (-3)^2)^{-1/2}$$
$$= 3(16)^{-1/2}$$
$$= 3\left(\tfrac{1}{\sqrt{16}}\right)$$
$$= \tfrac{3}{4}$$

1pt - $f'(-3)$

1pt - eq. line

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(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x+7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

$\lim_{x\to -3^-} g(x) = \lim_{x\to -3^-} f(x)$

$= \lim_{x\to -3^-} \sqrt{25-x^2}$

$= 4$

$\lim_{x\to -3^+} g(x) = \lim_{x\to -3^+} (x+7)$

$= -3+7$

$= 4$

$\lim_{x\to -3} g(x) = 4$

1pt - considers one-sided limits

$g(-3) = f(-3)$

$= 4$

Since $\lim_{x\to -3} g(x) = g(-3)$, g cont @ $x=-3$

1pt - answer w/ reason

(d) Find the value of $\int_0^5 x\sqrt{25-x^2}\,dx$.

$u = 25 - x^2$ $\quad u(0) = 25$, $u(5) = 0$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

$= \int_{25}^{0} x\sqrt{u} \cdot \frac{du}{-2x}$

$= -\frac{1}{2}\int_{25}^{0} u^{1/2}\,du$

2pts - antiderive

$= -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)\Big|_{25}^{0}$

$= -\frac{1}{3}\left(0^{3/2} - 25^{3/2}\right)$ ← ok, to stop here

1pt - answer

$= -\frac{1}{3}\left(-(\sqrt{25})^3\right)$

$= \frac{125}{3}$

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5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

↳ rate

$$\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = \frac{60}{5} = 12 \text{ grams/day}$$

1pt - uses $\frac{dB}{dt}$

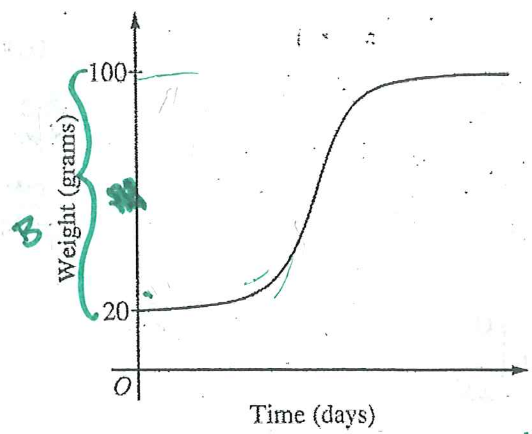
$$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = \frac{30}{5} = 6 \text{ grams/day}$$

1pt - answer w/ reason

Bird gaining weight faster when weighs 40 grams.

b/c $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



graph shows B btn 20 + 100

$\frac{d^2B}{dt^2}$ 20 (30) 100

$$\frac{d^2B}{dt^2} = \frac{1}{5}(-1)\frac{dB}{dt}$$

$$= -\frac{1}{5}\left(\frac{1}{5}(100-B)\right)$$

$$0 = -\frac{1}{25}(100-B) \rightarrow B = 100$$

1pt - $\frac{d^2B}{dt^2}$

1pt - explanation

Since $\frac{d^2B}{dt^2} < 0$ when B is (20,100), B should be concave down on (20,100), but the graph is concave up for some of the interval.

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Continue problem 5 on page 19.

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(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$\frac{dB}{dt} = \frac{1}{5}(100-B)$

$dB = \frac{\frac{1}{5}(100-B)\,dt}{(100-B)}$

1pt-separate
1pt-antiderive
1pt-"+C"

$\int \frac{1}{100-B}\,dB = \int \frac{1}{5}\,dt$

$u = 100-B$

$\frac{du}{dB} = -1$

$du = -dB$

$\int \frac{1}{u}\cdot -du = \frac{1}{5}t + C$

$-\ln|u| = \frac{1}{5}t + C$

$-\ln|100-B| = \frac{1}{5}t + C$

1pt initial condition

$-\ln|100-20| = \frac{1}{5}(0) + C$

$-\ln 80 = C$

$-\ln|100-B| = \frac{1}{5}t - \ln 80$

$\ln|100-B| = -\frac{1}{5}t + \ln 80$

$|100-B| = e^{-\frac{1}{5}t + \ln 80}$

$100-B = e^{-\frac{1}{5}t + \ln 80}$

$-B = e^{-\frac{1}{5}t + \ln 80} - 100$

1pt-solves for B

$\boxed{B = 100 - e^{-\frac{1}{5}t + \ln 80}}$

or $B = 100 - 80e^{-\frac{1}{5}t}$

OR

$\frac{dB}{20-\frac{1}{5}B} = \frac{(20-\frac{1}{5}B)\,dt}{20-\frac{1}{5}B}$

$\int \frac{1}{20-\frac{1}{5}B}\,dB = \int dt$

$\int \frac{1}{u}\cdot\frac{du}{-\frac{1}{5}} = t + C$

$u = 20 - \frac{1}{5}B$

$\frac{du}{dB} = -\frac{1}{5}$

$\frac{du}{-\frac{1}{5}} = dB$

$-5\int \frac{1}{u}\,du = t + C$

$-5\ln|u| = t + C$

$-5\ln|20-\frac{1}{5}B| = t + C$

$-5\ln|20-\frac{1}{5}20| = C$

$-5\ln 16 = C$

$-5\ln|20-\frac{1}{5}B| = t - 5\ln 16$

$\ln|20-\frac{1}{5}B| = \frac{t-5\ln 16}{-5}$

$|20-\frac{1}{5}B| = e^{\frac{t-5\ln 16}{-5}}$

$20-\frac{1}{5}B = e^{\frac{t-5\ln 16}{-5}}$

$-\frac{1}{5}B = e^{\frac{t-5\ln 16}{-5}} - 20$

$B = -5\left(e^{\frac{t-5\ln 16}{-5}} - 20\right)$

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6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

↳ velocity negative

$v(t) = \cos\left(\frac{\pi}{6}t\right)$

$\cos^{-1}(0) = \cos\left(\frac{\pi}{6}t\right)$

$\frac{6}{\pi} \cdot \frac{\pi}{2} = \cancel{\cos}\left(\frac{\pi}{6}t\right) \cdot \frac{6}{\pi}$ $\qquad$ $\frac{6}{\pi} \cdot \frac{3\pi}{2} = \cancel{\cos}\ \frac{\pi}{6}t \cdot \frac{6}{\pi}$

$3 = t$ $\qquad$ $9 = t$

$v(t)$: $+$ $-$ $+$

0 1 3 5 9 11 12

Particle moving left on $(3, 9)$ b/c $v(t) < 0$ on that interval.

1pt – $v(t) = 0$

1pt – interval

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

total distance traveled $= \int_0^6 |v(t)|\, dt$

1pt – answer

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(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

$\Rightarrow$ a(t) & v(t) same sign?

$a(t) = v'(t)$

$= -\sin(\frac{\pi}{6}t) \cdot \frac{\pi}{6}$

1pt – a(t)

$a(4) = -\sin(\frac{\pi}{6} \cdot 4) \cdot \frac{\pi}{6}$

$= -\frac{\pi}{6}\sin(\frac{2\pi}{3})$

< 0

$v(4) = \cos(\frac{\pi}{6} \cdot 4)$

$= \cos(\frac{2\pi}{3})$

< 0

Since $a(4) < 0$ and $v(4) < 0$, speed of particle is increasing

2pts – answer w/ reason

(d) Find the position of the particle at time $t = 4$.

$x(4) = -2 + \int_0^4 v(t)\,dt$

$= -2 + \int_0^4 \cos(\frac{\pi}{6}t)\,dt$

$u = \frac{\pi}{6}t$

$\frac{du}{dt} = \frac{\pi}{6}$

$\frac{6}{\pi}du = dt$

$u(0) = 0$

$u(4) = \frac{2\pi}{3}$

$= -2 + \int_0^{2\pi/3} \cos u \cdot \frac{6}{\pi}\,du$

1pt – antiderive

1pt – initial condition

$= -2 + \frac{6}{\pi}(\sin u)\Big|_0^{2\pi/3}$

$= -2 + \frac{6}{\pi}(\sin\frac{2\pi}{3} - \sin 0)$ ← ok to stop here

$= -2 + \frac{6}{\pi}(\frac{\sqrt{3}}{2} - 0)$

1pt – answer

$= -2 + \frac{3\sqrt{3}}{\pi}$

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