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1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G'(5) = -24.588$

↳ G(t)

1pt - $G'(5)$

The rate at which unprocessed gravel arrives at plant @ $t = 5$ hr is decreasing at 24.588 tons/hr^2

1pt - meaning w/ units

(t) neg. means (t) dec.

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(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

total amount of unprocessed gravel $= \int_0^8 G(t)\,dt$

$= 825.551$ tons.

1pt - integral

1pt - answer

A(t) = total amount unprocessed gravel

1:

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Continue problem 1 on page 5.

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$A(t)$ = amount unprocessed gravel

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

$A(t)$ inc? $A(t)$ dec?
$A'(t) > 0$? $A'(t) < 0$?

rate of unprocessed gravel = rate gravel arrives − rate gravel unprocessed

$$A'(t) = G(t) - 100$$
$$A'(5) = G(5) - 100$$
$$= -1.859$$

Amount of unprocessed gravel is decreasing @ $t = 5$ b/c $A'(5) < 0$

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

abs max of $A(t)$ (use $A'(t)$ from part c) ☺
candidates test → evaluate crit #s and endpts.

$0 \le t \le 8$

$A'(t) = 0$

$G(t) - 100 = 0$

$G(t) = 100$

$t = 4.923$

1pt - considers $A'(t) = 0$

$$A(4.923) = 500 + \int_0^{4.923} (G(t) - 100)\,dt = 635.376$$

$$A(8) = 500 + \int_0^{8} (G(t) - 100)\,dt = 525.551$$

$$A(0) = 500 + \int_0^{0} (G(t) - 100)\,dt = 500$$

1pt - answer
1pt - reason

Max amount of unprocessed gravel is 635.376 tons.

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2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

speed = $|v(t)|$

$|v(t)| = 2$

$v(t) = 2$ or $v(t) = -2$

@ $t = 3.128$ and $t = 3.473$

- 1pt - $|v(t)| = 2$

1pt - answer

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 (v(x))\,dx$

$= -9.207$

1pt - $s(t)$

1pt - $s(5)$

Continue problem 2 on page 7.

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(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

↳ v(t) changes signs

$v(t) = 0$

$t = 0.536$ and $t = 3.318$

1 pt – v(t) = 0

particle changes direction @ $t = 0.536$ and $t = 3.318$ b/c v(t) changes signs @ $t = 0.536$ and $t = 3.318$

2 pts – answers w/ reasons

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(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

↳ v(t) and a(t) same sign

↳ v(t) & a(t) diff. signs

$v(4) = -11.476$

$a(4) = v'(4)$

$= -22.296$

Speed inc @ $t = 4$ b/c $v(4) < 0$ and $a(4) < 0$

2 pts conclusion w/ reason

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