1 2 3 2

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

$$R'(2) = \frac{R(3) - R(1)}{3 - 1}$$

$$= \frac{950 - 1190}{3 - 1}$$

$$= -120 \text{ Liters/hr}^2$$

$\frac{\text{Liters/hr}}{\text{hr}}$ ... ☹

1pt : estimate

1pt : units

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

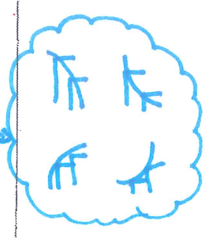
$$\int_0^8 R(t)\,dt \approx 1(1340) + 2(1190) + 3(950) + 2(740)$$

$$= 8050 \text{ Liters}$$

1pt : Left Riemann Sum

1pt : estimate

☺ ... Left rectangles



R is decreasing on (0,8),

∴, the estimate is an overestimate

1pt : over w/ reason

Continue problem 1 on page 5.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

total amount water in tank = initial amount + water in − water out

$= 50000 + \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt$

1pt: integral

$= 50000 + \int_0^8 W(t)\,dt - 8050$

$= 49786.195$

$= 49786$ Liters

1pt: estimate

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

will Rate pumped in = Rate removed ?

$W(t) = R(t)$?

$W(t) - R(t) = 0$?

$W(0) - R(0) = 48000$

$W(8) - R(8) = -618.476$

1pt: considers $W(t) - R(t)$

Yes, there is a time on $(0,8)$, by IVT,

since $W(0) - R(0) < 0$ and $W(8) - R(8) > 0$,

$\therefore W(t) - R(t) = 0$ on $(0,1)$

1pt: answer w/ explanation

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2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down? → $a(t)$ + $v(t)$ different signs?

↳ $a(t)$ + $v(t)$ same signs?

$v(4) = 2.979$

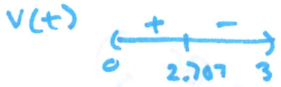
$a(4) = v'(4)$
$= -1.164$

2pts: conclusion w/ reason

Particle slowing down b/c $v(t) > 0$ and $a(t) < 0$ @ $t = 4$.

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

↳ $v(t)$ changes signs



Particle changes direction @ $t = 2.707$

b/c $v(t)$ changes signs @ $t = 2.707$

1pt: $t = 2.707$

1pt: reason

Continue problem 2 on page 7.

(c) Find the position of the particle at time $t = 0$.

$x(\cancel{t}) =$ initial position @t=4 $+ \int_4^t v(t)\,dt$

$x(0) = 2 + \int_4^0 v(t)\,dt$

$= -3.815$

1pt: integral
1pt: uses initial condition
1pt: answer

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

↳ $\int |v(t)|$

total distance $= \int_0^3 |v(t)|\,dt$

$= 5.301$

1pt: integral
1pt: answer

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