2018

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AP Calculus AB

Free-Response Questions

1. People enter a line for an escalator at a rate modeled by the function r given by

$$\text{derivative} \quad r(t) = \begin{cases} 44\left(\dfrac{t}{100}\right)^3\left(1 - \dfrac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are $\boxed{\text{20 people in line}}$ at time $t = 0$.

→ derivative → initial

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

↳ ∫ rate enter

$$\#\text{ people enter line} = \int_0^{300} r(t)\,dt$$

1pt: integral

$$= 270 \text{ people}$$

1pt: answer (units not required)

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

↳ # in line + # arrive − # leave

∫ rate enter − ∫ rate leave

$$\#\text{ people in line} = 20 + \int_0^{300} (r(t) - 0.7)\,dt$$

1pt: considers rate out

$$= 80 \text{ people}$$

1pt: answer (units not required)

Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

↳ start @ $t = 300$, so $r(t) = 0$, when $t > 300$

↳ no people = 0 = # in line + # enter − # leave

$$0 = 20 + \int_{300}^{t} (0 - 0.7)\, dx$$

$$0 = 20 + (-.7x)\Big|_{300}^{t}$$

$$0 = 20 + -.7t - (-.7(300))$$

$$t = 414.286 \text{ seconds}$$

1pt: answer

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

→ $t = ?$ → abs min ↳ $p = ?$

crit #s + end pts. in original eq.

people in line → $p(t) = 20 + \int_0^t (r(x) - 0.7)\, dx$

$$p'(t) = r(t) - 0.7$$

$$0 = r(t) - 0.7$$

$$r(t) = 0.7$$

$$t = 33.013$$

1pt: considers $r(t) - 0.7 = 0$

1pt: $t = 33.013$

$$P(33.013) = 20 + \int_0^{33.013} (r(t) - 0.7)\, dt$$

$$= 3.803$$

$P(0) = 20$

$P(300) = 80$

@ $t = 33.013$ sec, # of people in line is a minimum.

# of people in line @ $t = 33.013$ is 4 people.

1pt: justification

1pt: answers

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2. A particle moves along the x-axis with velocity given by $v(t) = \frac{10\sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$a(t) = v'(t)$

$a(3) = v'(3)$

$= -2.118$

1pt : answer

(b) Find the position of the particle at time $t = 3$.

$x(3) = x(0) + \int_0^3 v(t)\,dt$

$= -5 + \int_0^3 v(t)\,dt$

$= -1.760$

1pt : $\int_0^3 v(t)\,dt$

1pt : uses initial condition

1pt : answer

Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 2.844$

$\int_0^{3.5} |v(t)|\,dt = 3.737$

1 pt : answers

$\int_0^{3.5} v(t)\,dt$ is the change in position of particle from ~~t~~ $t=0$ to $t=3.5$.

$\int_0^{3.5} |v(t)|\,dt$ is the total distance the particle travels from $t=0$ to $t=3.5$

2 pt : interpretations

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$v_1 = v_2$

$v_2(t) = x_2'(t)$
$= 2t - 1$

$v_1(t) = v_2(t)$

$$\frac{10\sin(0.4t^2)}{t^2 - t + 3} = 2t - 1$$

$t = 1.571$

1 pt : sets $v(t) = x_2'(t)$

1 pt : answer

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