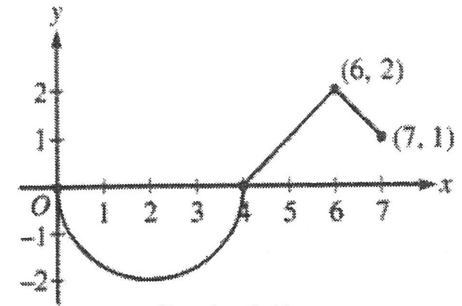
2022

AP® CollegeBoard

AP® Calculus AB

Free-Response Questions

Answer QUESTION 3 parts (a) and (b) on this page.



Graph of f'

$f(b) = \text{initial condition} + \int_a^b f'(x)\,dx$

Response for question 3(a)

1pt: area of either $\int_0^4 f'(x)dx$ OR $\int_4^5 f'(x)dx$

$$f(0) = f(4) + \int_4^0 f'(x)\,dx$$
$$= 3 - \int_0^4 f'(x)\,dx$$
$$= 3 - \left(-\tfrac{1}{2}\pi(2)^2\right)$$

← ok to stop here

$$= 3 + 2\pi$$

$$f(5) = f(4) + \int_4^5 f'(x)\,dx$$
$$= 3 + \tfrac{1}{2}(1)(1)$$

← ok to stop here

$$= \tfrac{7}{2} \text{ or } 3.5$$

1pt: $f(0)$
1pt: $f(5)$

pt of inf for f → f" changes signs
OR slope of f' changes signs
OR f' inc to dec, f' dec to inc

Response for question 3(b)

slopes of f' = f"

$- \;|\; + \;|\; + \;|\; -$
$\quad 2 \quad 4 \quad 6 \quad 7$

1pt: answer
1pt: reason

f has inf pts @ x = 2 and x = 6

b/c slopes of f' change signs @ x = 2 and x = 6.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 3 parts (c) and (d) on this page.

Response for question 3(c)

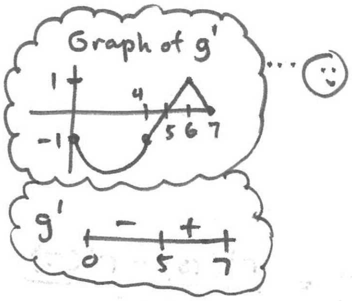
$g(x) = f(x) - x$

$g'(x) = f'(x) - 1$

$0 = f'(x) - 1$

$1 = f'(x)$

$x = 5, x = 7$



1pt: interval (as (0,5), [0,5], or [0,5))

g is dec on (0,5) b/c $g' < 0$ on (0,5)

1pt: reason

Response for question 3(d)

abs min → candidates test

$g(0) = f(0) - 0$
$= 3 + 2\pi$

1pt: considers $g'(x) = 0$

$g(5) = f(5) - 5$
$= 3.5 - 5$
$= -1.5$

$g(7) = f(7) - 7$
$= 3 + \int_5^7 f'(x)\,dx - 7$
$= 3 + \frac{1}{2}(2)(2) + 1 + \frac{1}{2}(1)(1) - 7$
$= -3 + 2 + \frac{1}{2}$
$= -\frac{1}{2}$

1pt: answer w/ justification

abs min value of g is -1.5

Answer QUESTION 4 parts (a) and (b) on this page.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	−6.1	−5.0	−4.4	−3.8	−3.5

Response for question 4(a)

$$r''(8.5) = \frac{r'(10) - r'(7)}{10-7}$$

$$= \frac{-3.8 - (-4.4)}{10-7}$$

$$= 0.2 \text{ cm/day}^2$$

1pt: $r''(8.5)$ w/ work

1pt: units

Response for question 4(b)

IVT ···

$r'(t)$ is cont on $[0,3]$ b/c r is twice-diff'able

$r'(0) = -6.1$
$r'(3) = -5.0$ } -6 is btn -6.1 and -5.0

1pt: betweenness $r'(0) < -6 < r'(3)$

∴, there is a time t on $(0,3)$ for which $r'(t) = -6$.

1pt: conclusion using IVT.

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\int_0^{12} r'(t)\,dt \approx 2(-3.5) + 3(-3.8) + 4(-4.4) + 3(-5.0)$$

← ok to stop here

$$\approx -51$$

1pt: forms of right sum

1pt: answer

Response for question 4(d)

$\frac{dh}{dt} = -2$ cm/day

$\frac{dV}{dt} = ?$ @ $t = 3$ days

$r = 100$ cm

$h = 50$ cm

1pt: product rule

1pt: chain rule

1pt: answer

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(h \cdot 2r \frac{dr}{dt} + r^2 \frac{dh}{dt} \right)$$

$$= \frac{1}{3}\pi\left(50 \cdot 2 \cdot 100(-5) + 100^2(-2)\right)$$

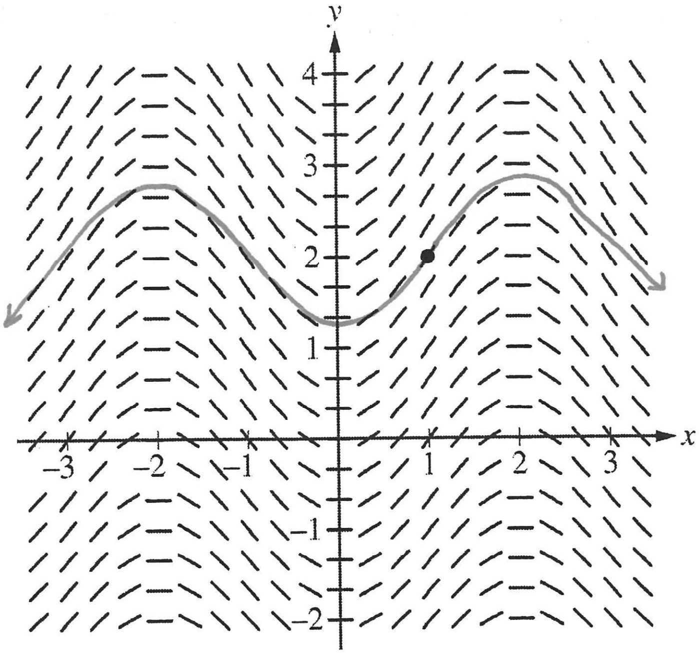
← ok to stop here

$$= -\frac{70000}{3}\pi$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)



1pt : solution

Response for question 5(b)

$y - y_1 = m(x - x_1)$

$\frac{dy}{dx}\Big|_{(1,2)} = \frac{1}{2}\sin(\frac{\pi}{2})\sqrt{2+7}$

$= \frac{1}{2}(1)(3)$

$= \frac{3}{2}$

$y - 2 = \frac{3}{2}(x - 1)$

1pt: tangent line equation

$f(0.8) \approx \frac{3}{2}(0.8 - 1) + 2$ ← ok to stop here.

$\approx \frac{3}{2}(-0.2) + 2$

$\approx -0.3 + 2$

≈ 1.7

1pt: approximation

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

The approx for part b is an underestimate for f(0.8) b/c $f'' > 0$ on $[-1, 1]$

1pt: answer w/ reason

Response for question 5(d)

1pt: separates variables

1pt: one correct antiderivative

1pt: other correct antiderivative

1pt: "+C" and initial condition

1pt: solves for y.

$$\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$$

$$\int \frac{1}{\sqrt{y+7}}\,dy = \int \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$$

$u = y+7$

$du = dy$

$u = \frac{\pi}{2}x$

$du = \frac{\pi}{2}dx$

$\frac{2}{\pi}du = dx$

$$\int u^{-1/2}\,du = \frac{1}{2}\cdot\frac{2}{\pi}\int \sin u\,du$$

$$2u^{1/2} = -\frac{1}{\pi}\cos u + C$$

$$2\sqrt{y+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + C \rightarrow 2\sqrt{y+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + 6$$

$$2\sqrt{2+7} = -\frac{1}{\pi}\left(\cos\frac{\pi}{2}\right) + C$$

$$6 = C$$

$$\sqrt{y+7} = \frac{-\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right)+6}{2}$$

$$y+7 = \left(\frac{-\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right)+6}{2}\right)^2$$

ok to stop here $\rightarrow$ $y = \left(\frac{-\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right)+6}{2}\right)^2 - 7$

OR $y = \left(-\frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)+3\right)^2 - 7$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$V_p(t) = x_p'(t)$

$= -(-4e^{-t})$

$= 4e^{-t}$

1pt : answer

Response for question 6(b)

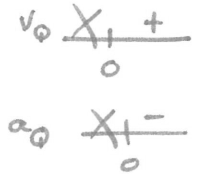
$V_\varphi(t) = t^{-2}$

$a(t)$ & $v(t)$ same/different signs

$a_\varphi(t) = V_\varphi'(t)$

$= -2t^{-3}$

$= \frac{-2}{t^3}$



1pt : $a_\varphi(t)$

1pt : considers signs of $a_\varphi(t)$ and $v_\varphi(t)$

Speed is dec for particle φ on $(0, \infty)$ b/c $V_\varphi(t) > 0$ and $a_\varphi(t) < 0$ on $(0, \infty)$

1pt : answer w/ reason

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$y_Q(t) = y_Q(1) + \int_1^t v_Q(x)\,dx$$

1pt: integral

$$= 2 + \int_1^t \frac{1}{x^2}\,dx$$

$$= 2 + \int_1^t x^{-2}\,dx$$

1pt: uses initial condition

$$= 2 - (x^{-1})\Big|_1^t$$

$$= 2 - \left(\frac{1}{t} - 1\right) \leftarrow \text{ok to stop here}$$

$$= 2 - \frac{1}{t} + 1$$

1pt: answer

$$= 3 - \frac{1}{t}$$

Response for question 6(d)

$x_P(t) = 6 - 4e^{-t}$ $y_Q(t) = 3 - \frac{1}{t}$

1pt: one correct limit

$\lim_{t\to\infty} (6 - 4e^{-t}) = 6$ $\lim_{t\to\infty} (3 - \frac{1}{t}) = 3$

$6 > 3 > 0$

$\therefore$, Particle P will be farther from the origin as $t \to \infty$.

1pt: answer w/reason