TEST #3

Non-Calculator

1. B
2. B
3. B
4. E
5. A
6. A
7. D
8. C
9. D
10. B
11. D
12. E
13. A
14. D
15. D
16. C
17. B
18. E
19. E
20. E
21. D
22. B
23. B
24. D
25. D
26. A
27. E
28. E

① $g'(x) = \frac{1}{8}x^3 - 10x$

$g'(4) = \frac{1}{8}(4)^3 - 10(4)$

$= \frac{64}{8} - 40$

$= 8 - 40$

$= -32$

$\boxed{B}$

② $2\int_0^{1/2} \frac{1}{\sqrt{1-(x)^2}}\,dx$

$= 2\sin^{-1}(x)\Big|_0^{1/2}$

$= 2\left[\sin^{-1}(1/2) - \sin^{-1}(0)\right]$

$= 2(\pi/6 - 0)$

$= \pi/3$

$\boxed{B}$

$f(x) = \sin^{-1}(x)$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

③ $\lim_{x\to 5} \frac{(x-5)(x+5)}{x-5} = \lim_{x\to 5}(x+5)$

$= 10$

$\boxed{B}$

④ $f'(x) = \frac{(x^3+7)(5x^4-1) - (x^5-x+2)(3x^2)}{(x^3+7)^2}$

$\boxed{E}$

⑤ $\lim_{h\to 0} \frac{5(\frac{1}{2}+h)^4 - 5(\frac{1}{2})^4}{h}$

definition of derivative!

$f'(\frac{1}{2}) = \lim_{h\to 0} \frac{5(\frac{1}{2}+h)^4 - 5(\frac{1}{2})^4}{h}$

where $f(x) = 5x^4$

$f'(x) = 20x^3$

$f'(\frac{1}{2}) = 20(\frac{1}{2})^3$

$= \frac{20}{8}$

$= \frac{5}{2}$

$\boxed{A}$

⑥ $\int x\sqrt{3x}\,dx$

$= \int x\sqrt{3}\sqrt{x}\,dx$

$= \sqrt{3}\int x\sqrt{x}\,dx$

$= \sqrt{3}\int x \cdot x^{1/2}\,dx$

$= \sqrt{3}\int x^{3/2}\,dx$

$= \sqrt{3}\left(\frac{2}{5}x^{5/2}\right) + C$

$= \frac{2\sqrt{3}}{5}x^{5/2} + C$

$\boxed{A}$

⑦ $\lim_{x\to 4} f(x) = f(4)$

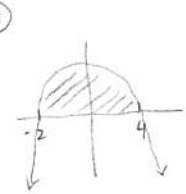
$\lim_{x\to 4} \frac{x^2-16}{x-4} = k$

$\lim_{x\to 4} \frac{(x-4)(x+4)}{x-4} = k$

$8 = k$

$\boxed{D}$

⑧



$8 + 2x - x^2 = 0$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4,\ x = -2$

Area $= \int_{-2}^{4} (8 + 2x - x^2)\,dx$

$\boxed{C}$

⑨ $f(x) = x^2\cos 2x$

$f'(x) = \cos 2x\,(2x) + x^2(-\sin 2x)\cdot 2$

'2' outside

$= 2x\cos 2x - 2x^2\sin 2x$

$\boxed{D}$

(10) $y(3) = 4(3)^3 - 7(3)^2$

$= 4 \cdot 27 - 7 \cdot 9$

$= 108 - 63$

$= 45$

$y' = 12x^2 - 14x$

$y'(3) = 12(3)^2 - 14(3)$

$= 12 \cdot 9 - 42$

$= 108 - 42$

$= 66$

$y - 45 = 66(x-3)$

$\boxed{B}$

(11) when $x = y$, $\frac{dy}{dx} = 0$

$\boxed{D}$

(12) MVT → f cont? yes

f diff'able? yes

$f'(c) = 6c - 5$

set equal

$\frac{f(b) - f(a)}{b - a}$

$\frac{f(5) - f(2)}{5 - 2}$

$\frac{51 - 3}{3}$

$\frac{48}{3}$

16

$6c - 5 = 16$

$6c = 21$

$c = \frac{7}{2}$

$\boxed{E}$

(13) abs max → highest y-value

$f'(x) = 4x - 7$

$0 = 4x - 7$

$x = 7/4$ crit #

check crit # + endpts for highest y-value

$f(7/4) = 2(7/4)^2 - 7(7/4) - 10 = \frac{49}{8} - \frac{49}{4} - 10 = \frac{49}{8} - \frac{98}{8} - \frac{80}{8}$

$f(-1) = 2(-1)^2 - 7(-1) - 10 = 2 + 7 - 10 = -1$ highest y-value.

$f(3) = 2(3)^2 - 7(3) - 10 = 12 - 21 - 10 = -19$

$\boxed{A}$

(14) $y(3x^2) + x^3 \frac{dy}{dx} + y^3(1) + x \cdot 3y^2 \frac{dy}{dx} = 0$

$x^3 \frac{dy}{dx} + 3xy^2 \frac{dy}{dx} = -3x^2y - y^3$

$\frac{dy}{dx}(x^3 + 3xy^2) = -3x^2y - y^3$

$\frac{dy}{dx} = -\frac{(3x^2y + y^3)}{x^3 + 3xy^2}$

$\boxed{D}$

(15) $f(x) = (1 + x^{1/2})^{1/2}$

$f'(x) = \frac{1}{2}(1 + x^{1/2})^{-1/2}\left(\frac{1}{2}x^{-1/2}\right)$

$= \frac{1}{4}x^{-1/2}(1 + x^{1/2})^{-1/2}$

$= \frac{1}{4\sqrt{x}\sqrt{1 + \sqrt{x}}}$

$\boxed{D}$

(16) $\int 7xe^{3x^2}\,dx$

$u = 3x^2$

$\frac{du}{dx} = 6x$

$\frac{du}{6x} = dx$

$= \int 7\cancel{x}e^u \cdot \frac{du}{6\cancel{x}}$

$= \frac{7}{6}\int e^u\,du$

$= \frac{7}{6}e^u + C$

$= \frac{7}{6}e^{3x^2} + C$

$\boxed{C}$

(17) $18x + 32y\frac{dy}{dx} = 0$

$18(2) + 32(-1)\frac{dy}{dx} = 0$

$36 - 32\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{36}{32}$

$\frac{dy}{dx} = \frac{9}{8}$

$y + 1 = \frac{9}{8}(x - 2)$

$8y + 8 = 9(x - 2)$

$8y + 8 = 9x - 18$

$0 = 9x - 8y - 26$ $\boxed{B}$

(18) $a(t) = s''(t)$

$s'(t) = 3t^2 - 12t + 9$

$s''(t) = 6t - 12$

$s''(4) = 6(4) - 12$

$= 12$

$\boxed{E}$

(19) $f'(x) = 3^{\pi x} \cdot \pi \cdot \ln 3$

$= \pi \ln 3 \cdot 3^{\pi x}$

$\boxed{E}$

(20) avg value $= \frac{1}{e-1}\int_1^e \frac{1}{x}\,dx$

$= \frac{1}{e-1}(\ln|x|)\Big|_1^e$

$= \frac{1}{e-1}(\ln e - \ln 1)$

$= \frac{1}{e-1} \cdot (1 - 0) = \frac{1}{e-1}$

$\boxed{E}$

(21) $f(x) = (\sin x)^2$

$f'(x) = 2\sin x \cos x$

$f''(x) = \cos x\,(2\cos x) + 2\sin x(-\sin x)$

$= 2\cos^2 x - 2\sin^2 x$

$= 2(\cos x)^2 - 2(\sin x)^2$

$f'''(x) = 4\cos x \cdot -\sin x - 4\sin x \cdot \cos x$

$= -4\sin x\cos x - 4\sin x\cos x$

$= -8\sin x\cos x$

$= -4(2\sin x\cos x)$

$= -4\sin 2x$

double angle formula!

$\boxed{D}$

(22) normal $\perp$ to tangent line

$\frac{dy}{dx} = 1 + -\sin(xy)\cdot\left[y\cdot(1) + x\cdot\frac{dy}{dx}\right]$

$\frac{dy}{dx} = 1 - \sin(1\cdot 0)\left[1(1) + 0\cdot\frac{dy}{dx}\right]$

$\frac{dy}{dx} = 1 - 0$

$\frac{dy}{dx} = 1$

slope of normal $= -\frac{1}{1} = -1$

$\boxed{B}$

(23) $\int e^x e^{3x}\,dx = \int e^{4x}\,dx$

$u = 4x$

$\frac{du}{dx} = 4$

$\frac{du}{4} = dx$

$= \int e^u \cdot \frac{du}{4}$

$= \frac{1}{4}\int e^u\,du$

$= \frac{1}{4}e^u + C$

$= \frac{1}{4}e^{4x} + C$ $\boxed{B}$

(24) $\lim_{x\to 0}\frac{(\tan 2x)^3}{x^3} = \lim_{x\to 0}\frac{\left(\frac{\sin 2x}{\cos 2x}\right)^3}{x^3}$

$= \lim_{x\to 0}\frac{(\sin 2x)^3}{x^3}\cdot\frac{1}{(\cos 2x)^3}$

$= \lim_{x\to 0}\left(\frac{2\sin 2x}{2x}\right)^3\cdot\frac{1}{(\cos 2x)^3}$

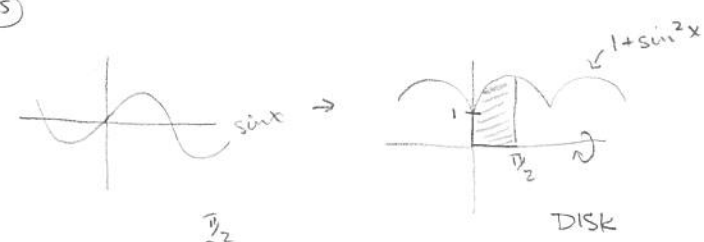
$= \lim_{x\to 0}(2)^3\cdot\frac{1}{(\cos 2x)^3}$

$= 8\cdot 1$

$= 8$

$\boxed{D}$

(25)



DISK

$V = \pi\int_0^{\pi/2}(1+\sin^2 x)^2\,dx$

$\boxed{D}$

(26) $\frac{dy}{dx} = 4\left(\frac{x^3-2}{2x^5-1}\right)^3\left(\frac{(2x^5-1)(3x^2)-(x^3-2)(10x^4)}{(2x^5-1)^2}\right)$

$\left.\frac{dy}{dx}\right|_{x=1} = 4\left(\frac{-1}{1}\right)^3\left(\frac{1(3)-(-1)(10)}{(1)^2}\right)$

$= 4(-1)(13)$

$= -52$

$\boxed{A}$

(27) $\int x(5-x)^{1/2}\,dx$

$u = 5-x$

$\frac{du}{dx} = -1$

$-du = dx$

Solve for x

$x = 5-u$

$= \int x(u^{1/2})\cdot -du$

$= -\int(5-u)u^{1/2}\,du$

$= -\int(5u^{1/2} - u^{3/2})\,du$

$= -\left(5\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$

$= -\frac{10}{3}(5-x)^{3/2} + \frac{2}{5}(5-x)^{5/2} + C$

$\boxed{E}$

(28) $\frac{dy}{dx} = \frac{x^3+1}{y}$

$\int y\,dy = \int(x^3+1)\,dx$

$\frac{1}{2}y^2 = \frac{1}{4}x^4 + x + C$

$\frac{1}{2}(2)^2 = \frac{1}{4}(1)^4 + 1 + C$

$2 = \frac{1}{4} + 1 + C$

$3/4 = C$

$\frac{1}{2}y^2 = \frac{1}{4}x^4 + x + 3/4$

$\frac{1}{2}y^2 = \frac{1}{4}(2)^4 + 2 + 3/4$

$\frac{1}{2}y^2 = 4 + 2 + 3/4$

$\frac{1}{2}y^2 = \frac{27}{4}$

$y^2 = \frac{27}{2}$

$y = \pm\sqrt{\frac{27}{2}}$

$\boxed{E}$