PreCal Review for Unit 8 DATE:________________

<u>NON-Calculator</u>

1) *Let* $\boldsymbol{a} = \langle -4, \frac{1}{2} \rangle$ and $\boldsymbol{b} = \langle \frac{2}{3}, -1 \rangle$. Find $4\boldsymbol{a} - 3\boldsymbol{b}$. Put in Linear Combination form.

$4a - 3b = 4\langle -4, \frac{1}{2} \rangle - 3\langle \frac{2}{3}, -1 \rangle$

$= \langle -16, 2 \rangle + \langle -2, +3 \rangle$

$= \langle -18, 5 \rangle$

$\boxed{-18i + 5j}$

2) Given Q= (7,2) and P= (-1,-2). Find the magnitude of the vector $\overrightarrow{PQ}$.

$(x_1, y_1) \quad (x_2, y_2)$

$v = \langle x_2 - x_1, y_2 - y_1 \rangle$

$\overrightarrow{PQ} = \langle 7 - -1, 2 - -2 \rangle$

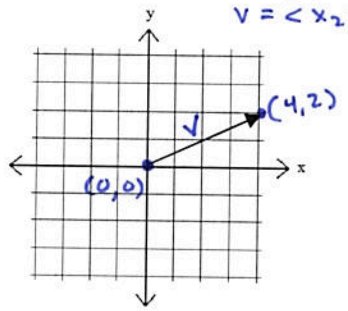
$= \langle 8, 4 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(+8)^2 + (+4)^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80}$

$\boxed{|\overrightarrow{PQ}| = 4\sqrt{5}}$



3) Find the component form of the vector.

$\boxed{\vec{v} = \langle 4, 2 \rangle}$

4) *Find the Unit Vector in the direction of*: $\boldsymbol{w} = \langle -15, 8 \rangle$.

$|\vec{w}| = \sqrt{(-15)^2 + 8^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289} = 17$

unit vector $= \frac{\langle -15, 8 \rangle}{17} = \boxed{\langle \frac{-15}{17}, \frac{8}{17} \rangle}$

5) *Find the Unit Vector in the direction of*: $\boldsymbol{w} = \langle 9, 3 \rangle$

$|\vec{w}| = \sqrt{9^2 + 3^2}$

$= \sqrt{81 + 9}$

$= \sqrt{90} = 3\sqrt{10}$

unit vector $= \frac{\langle 9, 3 \rangle}{3\sqrt{10}} = \boxed{\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \rangle}$ or $\boxed{\langle \frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10} \rangle}$

6) State & Verify 2 vectors one in the 2nd Q and one in the 3rd Q that are orthogonal.

$\rightarrow u \cdot v = 0$

$\vec{u} = \langle -3, -1 \rangle$

$\vec{v} = \langle -1, 3 \rangle$

$u \cdot v = -3(-1) + -1(3)$

$= 0$

$\therefore$, $\vec{u}$ and $\vec{v}$ are orthogonal

7) *Find the dot product of* $\boldsymbol{u}$ *and* $\boldsymbol{v}$. $\boldsymbol{u} = \langle \frac{2}{3}, -4 \rangle$ *and* $\boldsymbol{v} = \langle -2, \frac{2}{5} \rangle$.

$\vec{u} \cdot \vec{v} = \frac{2}{3}(-2) + -4(\frac{2}{5})$

$= \frac{-4}{3} - \frac{8}{5}$

$\frac{-20}{15} - \frac{24}{15} = \boxed{-\frac{44}{15}}$

8) *Find the dot product of* $\boldsymbol{u}$ *and* $\boldsymbol{v}$. $\boldsymbol{u} = -5\langle 1,0 \rangle + 2\langle 0,1 \rangle$ *and* $\boldsymbol{v} = 7\langle 1,0 \rangle - 9\langle 0,1 \rangle$.

$\vec{u} = \langle -5, 2 \rangle \qquad \vec{v} = \langle 7, -9 \rangle$

$\vec{u} \cdot \vec{v} = -5(7) + 2(-9) = -35 - 18 = \boxed{-53}$

9) Find the work done by a crane lifting a 585 lb. girder 72 ft.

$W = \vec{F} \cdot d$

$W = (585)(72) = \boxed{42120 \text{ ft} \cdot \text{lb}}$

10) Find the inverse of A, if A has an inverse.

$A = \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{\det A} \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}$

$= \frac{1}{5(4) - (-2)(1)} \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}$

$\frac{1}{22} \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}$

$\begin{bmatrix} 5/22 & 2/22 \\ -1/22 & 4/22 \end{bmatrix}$

$\boxed{A^{-1} = \begin{bmatrix} 5/22 & 1/11 \\ -1/22 & 2/11 \end{bmatrix}}$

Calculator

11) *Given* $Q = (11, -12)$ & $P = (-5, 4)$. *Find the component form of vector* $\overrightarrow{PQ}$.

$\overrightarrow{PQ} = \langle 11 - -5, -12 - 4 \rangle$

$\boxed{\overrightarrow{PQ} = \langle +16, -16 \rangle}$

(x_1, y_1) (x_2, y_2)

$v = \langle x_2 - x_1, y_2 - y_1 \rangle$

12) *Let* $\boldsymbol{v} = \langle -1, 1 \rangle$ *and* $\frac{1}{2}\boldsymbol{u} - 6\boldsymbol{v} = \langle 7, 4 \rangle$. *Find* $\boldsymbol{u}$.

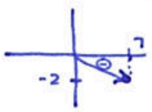
$\frac{1}{2}u - 6\langle -1, 1 \rangle = \langle 7, 4 \rangle$

$\frac{1}{2}u + \langle 6, -6 \rangle = \langle 7, 4 \rangle$

$\frac{1}{2}u = \langle 1, 10 \rangle$

$\boxed{\vec{u} = \langle 2, 20 \rangle}$

13) *Find the direction angle of the vector* $\boldsymbol{u} = \langle 7, -2 \rangle$



$|\vec{u}| = \sqrt{7^2 + (-2)^2}$
$= \sqrt{53}$

$\sin\theta = \frac{-2}{\sqrt{53}}$

$\theta = \sin^{-1}\left(\frac{-2}{\sqrt{53}}\right) = -15.945°$

$\theta = 360 - 15.945°$

$\boxed{\theta = 344.055°}$

14) *Find the Unit vector in the direction of* $\boldsymbol{v} = \langle -9, -11 \rangle$.

unit vector $= \frac{\vec{v}}{|\vec{v}|} = \frac{\langle -9, -11 \rangle}{\sqrt{(-9)^2 + (-11)^2}} = \frac{\langle -9, -11 \rangle}{14.213} = \boxed{\langle -.633, -.774 \rangle}$

15) *Find the angle between the 2 vectors,* $\boldsymbol{u} = \langle 6, -1 \rangle$ *and* $\boldsymbol{v} = \langle 2, 12 \rangle$.

$\cos\theta = \frac{u \cdot v}{|u||v|}$

$|u| = \sqrt{6^2 + (-1)^2}$
$|v| = \sqrt{2^2 + 12^2}$

$\cos\theta = \frac{6(2) + (-1)(12)}{\sqrt{37}\sqrt{148}}$

$\cos\theta = \frac{0}{\sqrt{37}\sqrt{148}}$

$\boxed{\theta = 90°}$

when $u \cdot v = 0$, vectors $\perp$ (orthogonal)

16) *Find the angle between the 2 vectors,* $\boldsymbol{u} = \langle 2, 2 \rangle$ *and* $\boldsymbol{v} = \langle -1, -4 \rangle$.

$\cos\theta = \frac{u \cdot v}{|u||v|}$

$|u| = \sqrt{2^2 + 2^2}$
$|v| = \sqrt{(-1)^2 + (-4)^2}$

$\cos\theta = \frac{2(-1) + 2(-4)}{\sqrt{8}\sqrt{17}}$

$\cos\theta = \frac{-10}{\sqrt{8}\sqrt{17}}$

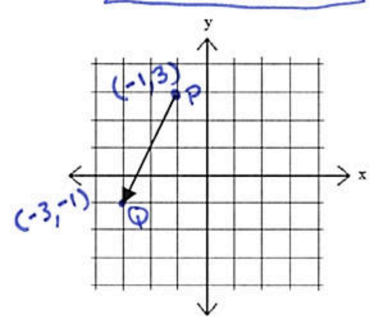
$\boxed{\theta = 149.036°}$

17) Find the component form of the vector.

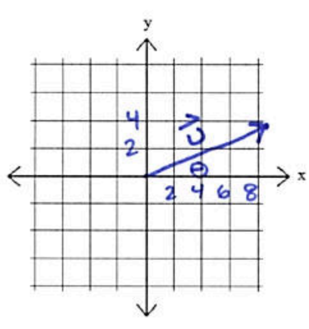
$P = (-1, 3)$, $Q = (-3, -1)$

$\overrightarrow{PQ} = \langle -3 - -1, -1 - 3 \rangle$

$\boxed{\overrightarrow{PQ} = \langle -2, -4 \rangle}$



18) Given $\boldsymbol{u} = \langle 8.2, 3.7 \rangle$. Draw $\boldsymbol{u}$ with magnitude and direction.



$|\vec{u}| = \sqrt{(8.2)^2 + (3.7)^2}$

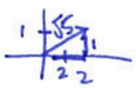
$\boxed{|\vec{u}| = 8.996}$

$\cos\theta = \frac{8.2}{8.996}$

$\theta = \cos^{-1}\left(\frac{8.2}{8.996}\right)$

$\boxed{\theta = 24.286°}$

19) Find the work done by a force ***F*** of 72 lbs. acting in the direction of $\langle 2,1\rangle$ in moving an object 5 feet along the x-axis starting at (0,0).

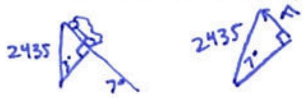


$W = \vec{F} \cdot d$

$= (72 \cdot \cos\theta)\, 5 \longrightarrow W = 72\left(\frac{2}{\sqrt{5}}\right) \cdot 5$

$\boxed{W = 321.994 \text{ ft} \cdot \text{lbs}}$

20) A car is parked on the side of a hill inclined at 7^0. The weight of the car is 2345 lbs. What force ***F*** is required to keep the car in place?



$\sin 7^\circ = \frac{\vec{F}}{2435}$

$\vec{F} = 2435 \sin 7^\circ$

$\vec{F} = \boxed{296.752 \text{ lbs}}$

21) Solve the system of equations

$$x + z + w = 2$$
$$x + y + z = 3$$
$$3x + 2y + 3z + w = 8$$

$$\text{rref}\left(\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 0 & 3 \\ 3 & 2 & 3 & 1 & 8 \end{array}\right]\right) = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + z + w = 2 \rightarrow x = 2 - z - w$

$y - w = 1 \longrightarrow y = 1 + w$

$z = z$

$w = w$

$\boxed{(2 - z - w, 1 + w, z, w)}$

22) Solve the system of equations

$$x + 2y + z = -1$$
$$x - 3y + 2z = 1$$
$$2x - 3y + z = 5$$

$$\text{rref}\left(\begin{bmatrix} 1 & 2 & 1 & -1 \\ 1 & -3 & 2 & 1 \\ 2 & -3 & 1 & 5 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 9/4 \\ 0 & 1 & 0 & -3/4 \\ 0 & 0 & 1 & -7/4 \end{bmatrix}$$

$\boxed{\left(\frac{9}{4}, -\frac{3}{4}, -\frac{7}{4}\right)}$

23) Find the partial fraction decomposition for $\dfrac{-x+10}{x^2+x-12} = \dfrac{-x+10}{(x+4)(x-3)}$

$\frac{-x+10}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$

$-x + 10 = Ax - 3A + Bx + 4B$

$-x + 10 = (A+B)x - 3A + 4B$

$-1 = A + B \qquad 10 = -3A + 4B$

$-3 = 3A + 3B$

$7 = 7B$

$1 = B$

$-1 = A + 1$

$-2 = A$

$\frac{-x+10}{x^2+x-12} = \boxed{\frac{-2}{x+4} + \frac{1}{x-3}}$

24) Find the partial fraction decomposition for $\dfrac{x^2-2x+1}{(x-2)^3}$

$\frac{x^2-2x+1}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$

$x^2 - 2x + 1 = A(x-2)^2 + B(x-2) + C$

$x^2 - 2x + 1 = A(x^2 - 4x + 4) + Bx - 2B + C$

$x^2 - 2x + 1 = Ax^2 - 4Ax + 4A + Bx - 2B + C$

$1 = A \qquad -2 = -4A + B \qquad 1 = 4A - 2B + C$

$-2 = -4(1) + B \qquad 1 = 4 - 4 + C$

$2 = B \qquad 1 = C$

$\boxed{\frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{(x-2)^3}}$

25) Represent the problem using an augmented matrix and solve the problem.

A florist makes cut flower arrangements for Mother's Day, involving roses, carnations, and lilies. The florist prices the arrangement at \$50 and roses cost \$3.50, carnations cost \$1.50, and lilies cost \$2. If the arrangement can have 24 flowers and there needs to be twice as many carnations as roses, how many of each type of flower is needed to make the arrangement?

$2x = y$

$x \rightarrow$ roses

$y \rightarrow$ carnations

$z \rightarrow$ lilies

$$3.50x + 1.50y + 2z = 50$$
$$x + y + z = 24$$
$$2x - y = 0$$

$$\begin{bmatrix} 3.50 & 1.50 & 2 & 50 \\ 1 & 1 & 1 & 24 \\ 2 & -1 & 0 & 0 \end{bmatrix}$$

$$\text{rref}\left(\begin{bmatrix} 3.5 & 1.5 & 2 & 50 \\ 1 & 1 & 1 & 24 \\ 2 & -1 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 12 \end{bmatrix}$$

$\boxed{\text{4 roses, 8 carnations, 12 lilies}}$