

## 1.2 Functions and Their Properties

### Symmetry, End Behavior of Functions

Target 1B: Analyze functions using specific properties

*Review of Prior Concepts*

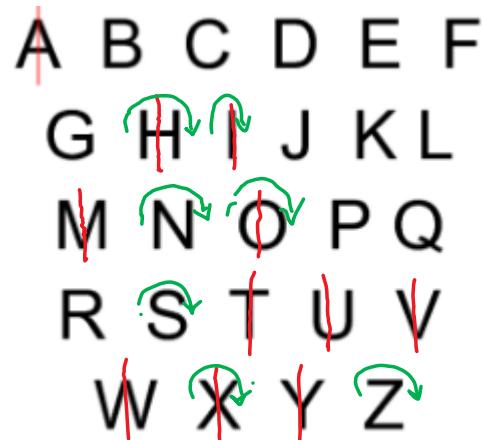
Which of the letters of the alphabet have vertical symmetry?

(Hint: A is one of them)

A, H, I, M, O, T, U, V, W, Y

Which have  $180^\circ$  rotational symmetry?  
*same image upside down*

H, I, N, O, S, X, Z

**More Practice****Symmetry**

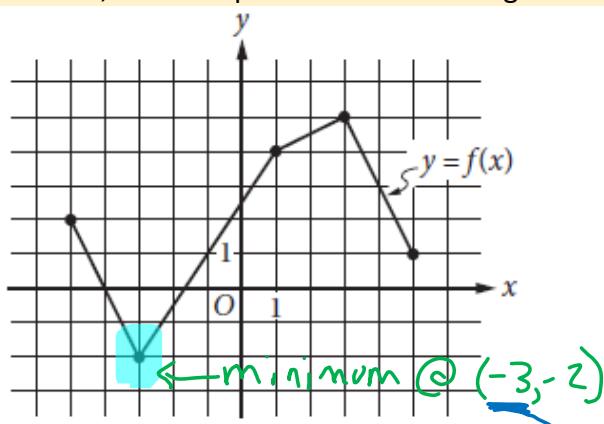
<http://gwydir.demon.co.uk/jo/symmetry/refsym.htm>

<https://www.khanacademy.org/math/geometry/transformations/transformations-symmetry/v/example-rotating-polygons>

**SAT Connection****Passport to Advanced Math**

**13.** Use function notation, and interpret statements using function notation.

Example:



The complete graph of the function  $f$  is shown in the  $xy$ -plane above. For what value of  $x$  is the value of  $f(x)$  at its minimum?

A) -5

B) -3

C) -2

D) 3

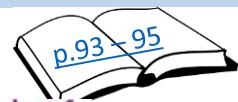
Solution

$$x = -3$$

## Symmetry

- Even Functions – (graphically) *symmetrical about y-axis*

– (numerically) *y-values for positive x is same as y-values for negative x*  
 – (algebraically)  $f(-x) = f(x)$

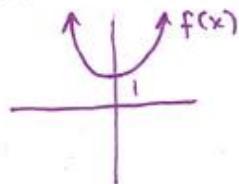


- Odd Functions – (graphically) *symmetrical about the origin*

– (numerically) *y-values for positive x is opposite as y-values for negative x*  
 – (algebraically)  $f(-x) = -f(x)$

Determine graphically whether the function is even, odd, or neither. Check algebraically.

ex:  $f(x) = 2x^4 + x^2 + 1$



$f(x)$  is even b/c  $f(x)$  is symmetrical about y-axis

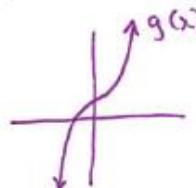
algebraically

$$\begin{aligned} f(-x) &= 2(-x)^4 + (-x)^2 + 1 \\ &= 2x^4 + x^2 + 1 \end{aligned}$$

\* replace  $-x$  for every  $x$

↑ same as  $f(x)$ ,  
so  $f(x)$  is even.

ex:  $g(x) = 2x^3 + x + 1$



$g(x)$  is neither odd nor even b/c  $g(x)$  is not symmetrical about y-axis nor origin.

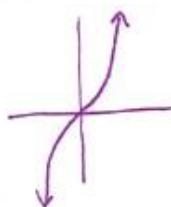
algebraically

$$\begin{aligned} f(-x) &= 2(-x)^3 + (-x) + 1 \\ &= -2x^3 - x + 1 \end{aligned}$$

\* replace  $-x$  for every  $x$

↑ not same as  $f(x)$

ex:  $h(x) = 2x^3 + x$



$h(x)$  is odd b/c  $h(x)$  is symmetrical about the origin.

algebraically

$$\begin{aligned} f(-x) &= 2(-x)^3 + (-x) \\ &= -2x^3 - x \end{aligned}$$

\* replace  $-x$  for every  $x$

↑ not same as  $f(x)$

$$\begin{aligned} f(-x) &= - (2x^3 + x) \\ &\sim \text{same as } f(x) \end{aligned}$$

\* try factoring out  $"-"$

$$\text{so, } f(-x) = -f(x)$$

$\therefore f(x)$  is odd.

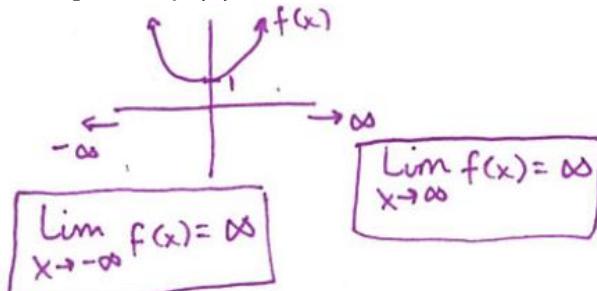
## End Behavior

End Behavior – what happens at the ends of the function.

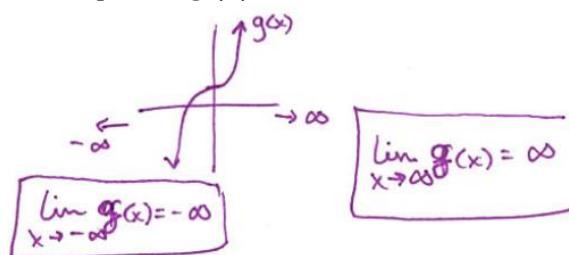
NOTATION:

Describe the end behavior of the function from the graph of the function.

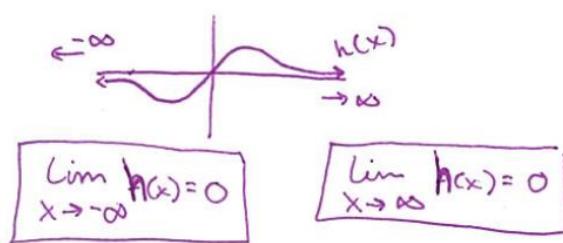
Example 4:  $f(x) = 2x^4 + x^2 + 1$



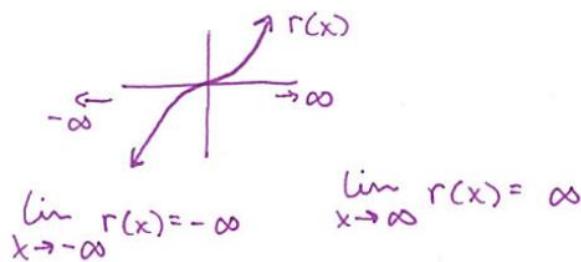
Example 5:  $g(x) = 2x^3 + x + 1$



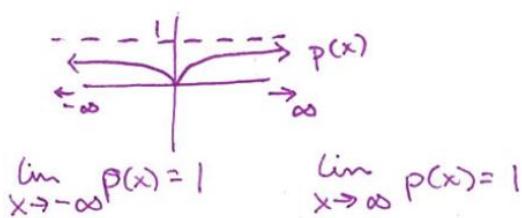
Example 6:  $h(x) = \frac{x}{x^2+2}$



Example 7:  $r(x) = \frac{x^3}{x^2+2}$



Example 8:  $p(x) = \frac{x^2}{x^2+2}$



Horizontal Asymptotes – occur when end behavior approaches a #, c. H.A. is @  $y = c$ .

NOTATION:  $\lim_{x \rightarrow \infty} f(x) = c$  or  $\lim_{x \rightarrow -\infty} f(x) = c$

since  $\lim_{x \rightarrow \infty} p(x) = 1$ ,

there is a horizontal asymptote  
@  $y = 1$

## More Practice

## Symmetry

<https://www.chilimath.com/algebra/intermediate/oef/even-and-odd-functions.html>

<https://www.youtube.com/watch?v=1LsJaR72UFM>

## End Behavior

<http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/14-tail-behavior-limits-at-infinity-02>

[https://www.youtube.com/watch?v=Krjd\\_vU4Uvg](https://www.youtube.com/watch?v=Krjd_vU4Uvg)

## Homework Assignment

p.98 #35,38,39,45,49,50,51,53

**SAT Connection****Solution**

**Choice B is correct.** The minimum value of the function corresponds to the  $y$ -coordinate of the point on the graph that is the lowest along the vertical or  $y$ -axis. Since the grid lines are spaced 1 unit apart on each axis, the lowest point along the  $y$ -axis has coordinates  $(-3, -2)$ . Therefore, the value of  $x$  at the minimum of  $f(x)$  is  $-3$ .

Choice A is incorrect;  $-5$  is the smallest value for an  $x$ -coordinate of a point on the graph of  $f$ , not the lowest point on the graph of  $f$ . Choice C is incorrect; it is the minimum value of  $f$ , not the value of  $x$  that corresponds to the minimum of  $f$ . Choice D is incorrect; it is the value of  $x$  at the maximum value of  $f$ , not at the minimum value of  $f$ .