

1.2 Functions and Their Properties

Vertical and Horizontal Asymptotes

Target 1B: Analyze functions using specific properties

Review of Prior Concepts

Identify any discontinuities for $f(x) = \frac{x^2+7x+10}{x^2-4x-12}$ and describe the type of discontinuity.

$$\begin{aligned}
 f(x) &= \frac{(x+5)(x+2)}{(x-6)(x+2)} \\
 &= \frac{(x+5)\cancel{(x+2)}}{\cancel{(x-6)}(x+2)} \\
 &\quad \uparrow \qquad \rightarrow \text{removable discontin.} \\
 \text{non removable} \quad @ x = 6 & \quad @ x = -2
 \end{aligned}$$

More Practice

Discontinuities

<http://www.ck12.org/Analysis/Discrete-and-Continuous-Functions/lesson/Continuity-and-Discontinuity-PCALC/>

<https://www.youtube.com/watch?v=2n5VzMFJQVY>

Vertical & Horizontal Asymptotes

RECALL:

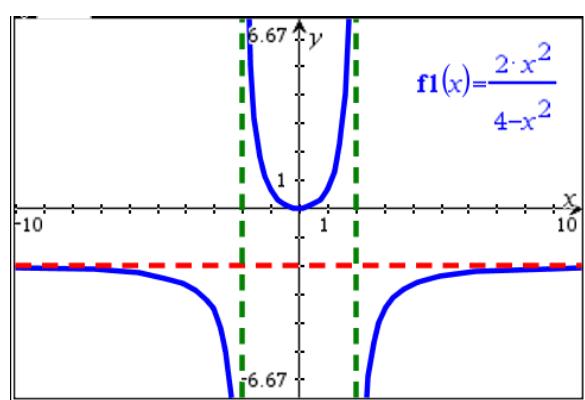
Vertical Asymptotes – non-removable discontinuity found from denominator set equal to zero.

Horizontal Asymptotes -- occur when end behavior approaches a #, c. H.A. is @ $y = c$.

NOTATION: $\lim_{x \rightarrow \infty} f(x) = c$ or $\lim_{x \rightarrow -\infty} f(x) = c$

Graph each function. Find vertical asymptotes algebraically & horizontal asymptotes graphically (if any).

Example 1: $g(x) = \frac{2x^2}{4-x^2}$

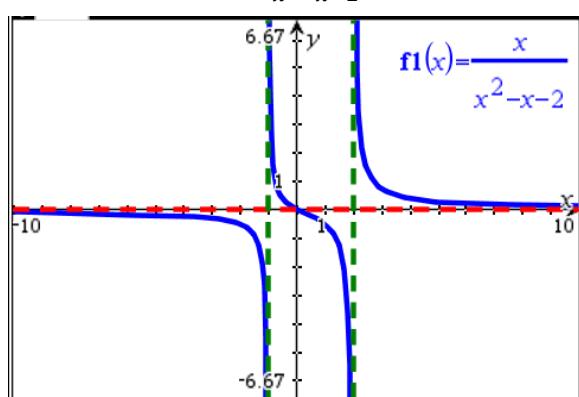


$$\begin{aligned}
 g(x) &= \frac{2x^2}{(2-x)(2+x)} \\
 2-x=0 \quad 2+x=0 \\
 2=x \quad x=-2 \\
 \text{V.A. } @ x = -2, x = 2
 \end{aligned}$$

H.A. @ $y = 2$ b/c

$$\lim_{x \rightarrow \infty} g(x) = 2, \lim_{x \rightarrow -\infty} g(x) = 2$$

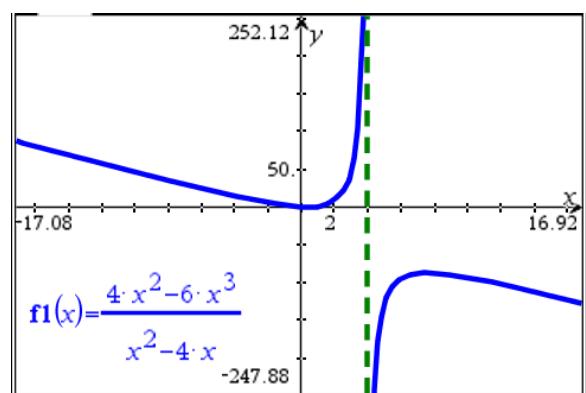
Example 2: $f(x) = \frac{x}{x^2 - x - 2}$



H.A. @ $y=0$ b/c

$$\lim_{x \rightarrow \infty} f(x) = 0, \lim_{x \rightarrow -\infty} f(x) = 0$$

Example 3: $h(x) = \frac{4x^2 - 6x^3}{x^2 - 4x}$



No H.A. b/c

$$\lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = \infty$$

$$f(x) = \frac{x}{(x-2)(x+1)}$$

$$x-2=0 \quad x+1=0 \\ x=2 \quad x=-1$$

V.A. @ $x=-1, x=2$

$$h(x) = \frac{x^2(4x-6)}{x(x-4)}$$

$$= \cancel{x}(x)(4x-6)$$

$$= \frac{\cancel{x}(4x-6)}{x-4}$$

$$x-4=0$$

$$x=4$$

V.A. @ $x=4$

@ $x=0$

Vertical Asymptotes

More Practice

<http://www.purplemath.com/modules/asymtote.htm>

<https://www.youtube.com/watch?v=h910Jbhzecl>

Homework Assignment

p.99 #57, 59, 62, 63, 64, 65, 73-76 all