

10.3 More on Limits

Target 9A,B,C: Evaluate a limit of a function analytically, graphically, & numerically

Target 9D: Calculate one-sided limits and two-sided limits

Review of Prior Concepts

If $f(x) = 2x^2 - 2$, find

$$\begin{aligned} \text{a) } f(-1) &= 2(-1)^2 - 2 \\ &= 2 - 2 \\ &= \boxed{0} \end{aligned}$$

$$\begin{aligned} \text{c) } f(x+h) &= 2(x+h)^2 - 2 \\ &= 2(x^2 + 2xh + h^2) - 2 \\ &= \boxed{2x^2 + 4xh + 2h^2 - 2} \end{aligned}$$

$$\begin{aligned} (x+h)^2 &= \\ &= (x+h)(x+h) \\ &= x^2 + xh + xh + h^2 \\ &= x^2 + 2xh + h^2 \end{aligned}$$

$$\begin{aligned} \text{b) } f(a) &= 2(a)^2 - 2 \\ &= \boxed{2a^2 - 2} \end{aligned}$$

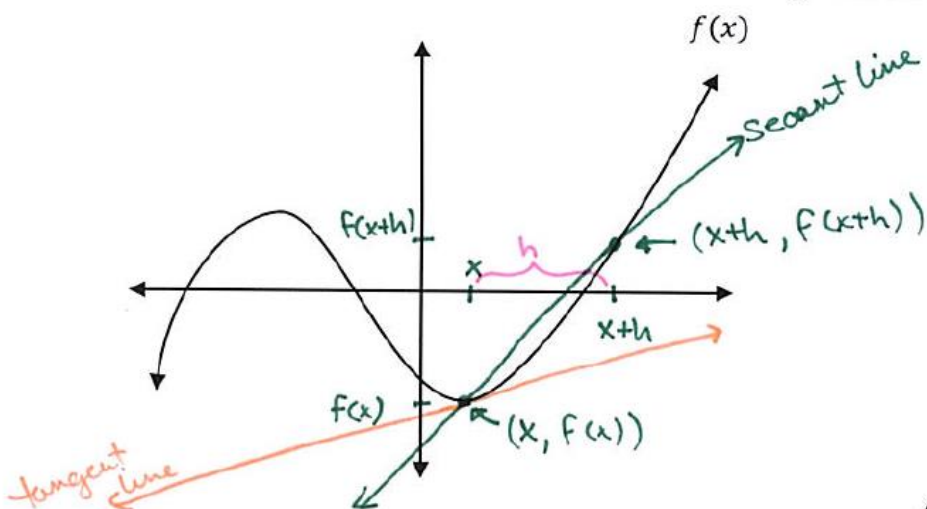
$$\begin{aligned} \text{d) } \frac{f(x+h) - f(x)}{h} &= \frac{2(x+h)^2 - 2 - (2x^2 - 2)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) - 2 - 2x^2 + 2}{h} \\ &= \frac{2x^2 + 2xh + h^2 - 2 - 2x^2 + 2}{h} \\ &= \frac{2xh + h^2}{h} \\ &= \frac{h(2x+h)}{h} \\ &= \boxed{2x+h} \end{aligned}$$

More Practice

Evaluating Functions

<http://www.mathsisfun.com/algebra/functions-evaluating.html>https://www.khanacademy.org/math/algebra/algebra-functions/evaluating-functions/e/functions_1<http://www.coolmath.com/algebra/15-functions/08-the-difference-quotient-01><https://youtu.be/E9YEUQR9NAU><https://youtu.be/1O5NEI8UuHM>

The Tangent Problem



secant line - intersects graph @ 2 pts

$$\begin{aligned} m_{\text{secant line}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{f(x+h) - f(x)}{x+h - x} \\ &= \frac{f(x+h) - f(x)}{h} \end{aligned}$$

Slope of secant line
= average rate of change
= average velocity



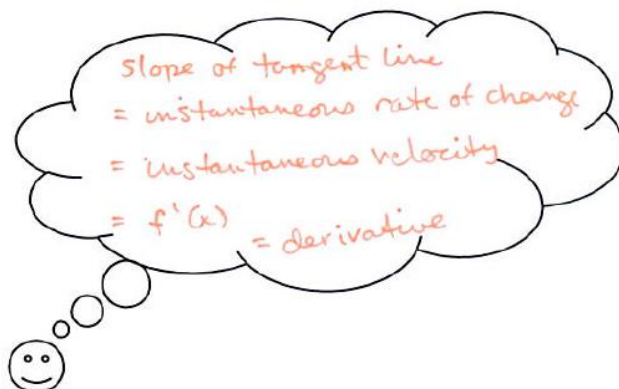
tangent line - touches graph @ one point

- moves the pt. $(x+h, f(x+h))$ really close to $(x, f(x))$
- \rightarrow so, h gets close to zero

$$m_{\text{tangent line}} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

say as: "f prime of x"



Examples:

1. Find the instantaneous rate of change @ $x = 2$ for $f(x) = 3x^2 - 2x + 1$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 2(x+h) + 1 - (3x^2 - 2x + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 1 - 3x^2 + 2x - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 1 - 3x^2 + 2x - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 2h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6x + 3h - 2)}{h} \end{aligned}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} (6x + 3h - 2) \\ &= 6x - 2 \\ f'(2) &= 6(2) - 2 \\ &= 10 \end{aligned}$$

2. Find the derivative of $f(x) = 8x - 4$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{8(x+h) - 4 - (8x - 4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{8x + 8h - 4 - 8x + 4}{h} \\ &= \lim_{h \rightarrow 0} \frac{8h}{h} \\ &= \lim_{h \rightarrow 0} 8 \end{aligned}$$

$$f'(x) = 8$$

instantaneous rate of change @ $x = 2$ is 10

3. Find $f'(x)$ for $f(x) = \sqrt{x}$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \quad \text{conjugate} \dots \text{😊} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\
 &= \lim_{h \rightarrow 0} \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}
 \end{aligned}$$

$f'(x) = \frac{1}{\sqrt{x+0} + \sqrt{x}}$
 $f'(x) = \frac{1}{2\sqrt{x}}$
 or $\frac{1}{2}x^{-1/2}$

More Practice

The Tangent Line

<http://clas.sa.ucsb.edu/staff/lee/secant,%20tangent,%20and%20derivatives.htm>

http://tutorial.math.lamar.edu/Classes/CalcI/Tangents_Rates.aspx

<https://youtu.be/qPOUPXIfEWU>

<https://youtu.be/uI9QLZGqV1A>

<https://youtu.be/ydHzk5zWd4I>

Homework Assignment

p. 803 #3,7,9,11,23,25,29
p. 803 #12,13,24,26,28,31