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Lagrange and Alternating Series Error Bound Practice

1. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \leq 3$ for all x in the closed interval $[1.5,2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

2. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{5^n \cdot n}$. When $x = 3.1$, the series converges to a value S. Use the first two terms of the series to approximate S. Use the alternating series error bound to show that this approximation differs from S by less than $\frac{1}{300,000}$.

3. Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$. Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$

4. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{nx^n}{2n^2+1}$ and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series. The first ten terms of the Maclaurin series for f are used to approximate $f(-1)$. Show that this approximation differs from $f(-1)$ by less than $\frac{1}{10}$.

5. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by $T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

a) Use $T(x)$ to find an approximation for $f(0)$.

b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \leq 6$ for all x in the closed interval $[0,2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (a) to explain why $f(0)$ is negative.