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10.1 & 10.2 Derivatives of Vectors and Parametric Functions

Recall from Pre-Calculus:

Vectors

$\langle x(t), y(t) \rangle$

Parametric Functions

$x(t) =$

$y(t) =$

Position

Vectors

$\langle x(t), y(t) \rangle$

Parametric Functions

$x(t) =$

$y(t) =$

Velocity

Vectors

$v(t) = \langle x'(t), y'(t) \rangle$

or

$v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$

Parametric Functions

$x'(t) =$ or $\frac{dx}{dt} =$

$y'(t) =$ $\frac{dy}{dt} =$

Acceleration

Vectors

$a(t) = \langle x''(t), y''(t) \rangle$

OR

$a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$

Parametric Functions

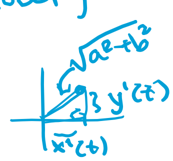
$x''(t) =$ or $\frac{d^2x}{dt^2} =$

$y''(t) =$ $\frac{d^2y}{dt^2} =$

Speed → magnitude of velocity
↳ length

Vectors

$v(t) = \langle x'(t), y'(t) \rangle$



Parametric Functions

$x'(t) =$

$y'(t) =$

speed

$$|v(t)| = \sqrt{(x'(t))^2 + (y'(t))^2}$$

or

$$= \sqrt{(dx/dt)^2 + (dy/dt)^2}$$

Slopes of Parametric Curves

Parametric Functions

$x(t) =$ $y(t) =$

$\frac{dx}{dt}$ $\frac{dy}{dt}$

Slope of a curve (slope of a tangent line):

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dx}$ ☺

Example:
Write the equation of the tangent line at $t = 4$ given the parametric function defined by $x(t) = \sqrt{t} - t$ and $y(t) = \frac{32}{t}$.

$y - \underbrace{y_1}_{8} = m(x - \underbrace{x_1}_{-2})$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$x(4) = \sqrt{4} - 4 = 2 - 4 = -2$

$y(4) = \frac{32}{4} = 8$

$\frac{dx}{dt} = \frac{1}{2}t^{-1/2} - 1$

$\frac{dy}{dt} = -32t^{-2}$

$\frac{dy}{dx} = \frac{-32t^{-2}}{\frac{1}{2}t^{-1/2} - 1}$

$\left.\frac{dy}{dx}\right|_{t=4} = \frac{\frac{-32}{(4)^2}}{\frac{1}{2\sqrt{4}} - 1} = \frac{\frac{-32}{16}}{\frac{1}{4} - 1} = \frac{-2}{-3/4} = -2 \cdot \frac{4}{-3} = \frac{8}{3}$

$y - y_1 = m(x - x_1)$

$\boxed{y - 8 = \frac{8}{3}(x + 2)}$

$$\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx} = \frac{d}{dx}\frac{(dy)}{(dx)} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

Example:

Find $\left.\frac{d^2y}{dx^2}\right|_{t=1}$ given the parametric function defined by $x(t) = t^3$ and $y(t) = t^2 + 2t$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+2}{3t^2}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{3t^2(2)-(2t+2)(6t)}{(3t^2)^2} = \frac{6t^2-12t^2-12t}{9t^4} = \frac{-6t^2-12t}{9t^4} = \frac{-6t(t+2)}{9t^4} = \frac{-2(t+2)}{3t^3}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{-2(t+2)}{3t^3}}{3t^2} = \frac{-2(t+2)}{3t^3}\cdot\frac{1}{3t^2} = \frac{-2(t+2)}{9t^5}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{-2(1+2)}{9(1)^5} = \frac{-6}{9} = \boxed{\frac{-2}{3}}$$