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Derivatives of Parametric Equations & Vectors Practice

1. If a particle moves in the xy-plane so that at any time $t > 0$, its position is $\langle \ln(t^2+5t), 3t^2 \rangle$, find the velocity vector at time $t = 2$.

$v(t) = \langle (2t+5) \cdot \frac{1}{t^2+5t}, 6t \rangle$

$= \langle \frac{2t+5}{t^2+5t}, 6t \rangle$

$v(2) = \langle \frac{2(2)+5}{2^2+5(2)}, 6(2) \rangle$

$\boxed{v(2) = \langle \frac{9}{14}, 12 \rangle}$

2. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?

$\rightarrow x'(t) = 0$ and $y'(t) = 0$

$x'(t) = 3t^2 - 3t - 18$

$0 = 3t^2 - 3t - 18$

$0 = 3(t^2 - t - 6)$

$0 = 3(t-3)(t+2)$

$t = 3, t = -2$

$y'(t) = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = 3(t-3)(t+1)$

$t = 3, t = 1$

$\boxed{\text{Particle is at rest when } t = 3 \text{ b/c } x'(3) = 0 \text{ and } y'(3) = 0}$

3. A particle moves in the xy-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8-t)(1-\cos t)$. Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

$25 = 5t + 3\sin t$

$t = 5.446$

$v(5.446) = \langle x'(5.446), y'(5.446) \rangle$

$\boxed{v(5.446) = \langle 7.008, -2.228 \rangle}$

4. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$. Find the equation of the tangent line at the point where $t = \frac{\pi}{4}$.

$x(\pi/4) = 2 - 3\cos \pi/4$

$= 2 - 3(\sqrt{2}/2)$

$= 2 - \frac{3\sqrt{2}}{2}$

$y(\pi/4) = 3 + 2\sin \pi/4$

$= 3 + 2\frac{\sqrt{2}}{2} = 3 + \sqrt{2}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{3\sin t}$

$\left.\frac{dy}{dx}\right|_{t=\pi/4} = \frac{2\cos \pi/4}{3\sin \pi/4}$

$= \frac{2(\sqrt{2}/2)}{3(\sqrt{2}/2)} = \frac{2}{3}$

$y - (3+\sqrt{2}) = \frac{2}{3}(x - (2 - \frac{3\sqrt{2}}{2}))$

$\boxed{y - 3 - \sqrt{2} = \frac{2}{3}(x - 2 + \frac{3\sqrt{2}}{2})}$

5. An object moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time $t \ge 0$ with $\frac{dx}{dt} = 1 + \tan t^2$ and $\frac{dy}{dt} = 3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t = 5$.

$a(5) = \langle \left.\frac{d^2x}{dt^2}\right|_{t=5}, \left.\frac{d^2y}{dt^2}\right|_{t=5} \rangle$

$\boxed{a(5) = \langle 10.178, 6.277 \rangle}$

speed

$|v(5)| = \sqrt{\left(\left.\frac{dx}{dt}\right|_{t=5}\right)^2 + \left(\left.\frac{dy}{dt}\right|_{t=5}\right)^2}$

$\boxed{|v(5)| = 28.083}$

6. A particle moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dy}{dt} = 2 + \sin e^t$. The derivative $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4,5)$ and the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

$\left.dy/dx\right|_{t=3} = \frac{\left.dy/dt\right|_{t=3}}{\left.dx/dt\right|_{t=3}}$

$1.8 = \frac{2 + \sin e^3}{\left.dx/dt\right|_{t=3}} \longrightarrow \left.dx/dt\right|_{t=3} = \frac{2 + \sin e^3}{-1.8}$

$\boxed{\left.dx/dt\right|_{\_\_} = -1.636}$