DATE: ___________

Arc Length (Parametric & Vectors)

1. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\pi/2} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

(B) $\int_0^{\pi/2} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\pi/2} \sqrt{9\cos^4 t + 3\sin^4 t}\, dt$

(D) $\int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 3\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\pi/2} \sqrt{\cos^6 t + \sin^6 t}\, dt$

$x = (\cos t)^3$ $\quad$ $y = (\sin t)^3$

$\frac{dx}{dt} = 3(\cos t)^2 \cdot -\sin t = -3\cos^2 t \sin t$ $\quad$ $\frac{dy}{dx} = 3(\sin t)^2 \cdot \cos t = 3\sin^2 t \cos t$

$L = \int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt$

$= \int_0^{\pi/2} \sqrt{9\cos^4 \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

2. A particle move in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909

(B) 3.062

(C) 6.884

(D) 9.016

(E) 47.393

$x'(t) = 2t$

$y'(t) = \cos(4t) \cdot 4 = 4\cos(4t)$

speed $= |v(t)| = \sqrt{(x'(t))^2 + (y'(t))^2}$

$|v(3)| = \sqrt{(x'(3))^2 + (y'(3))^2}$

$= 6.884$

3. For $t \ge 0$, a particle is moving along a curve that that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position (1,5). It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$. Find the speed of the particle at time $t = 4$. Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

speed $= |v(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$|v(4)| = \sqrt{\left(\left.\frac{dx}{dt}\right|_{t=4}\right)^2 + \left(\left.\frac{dy}{dt}\right|_{t=4}\right)^2}$

$= 0.575$

distance $= \int_2^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= 0.651$