DATE: _______________

AP FRQs

1991 BC4 Part (d)

Let $F(x) = \int_1^{2x} \sqrt{t^2 + t}\, dt$. Find the length of the curve $y = F(x)$ for $1 \leq x \leq 2$.

1992 BC3 Part (c)

At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$. Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

1993 BC2 Part (b)

The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$. Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

2002B BC1 Part (d)

A particle moves in the xy-plane so that its position at any time t, for $-\pi \leq t \leq \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$. Is the distance traveled by the particle from $t = -\pi$ and $t = \pi$ greater than 5π? Justify your answer.

Solutions

1991 BC4 Part (d)

(d) $L = \int_1^2 \sqrt{1+\left(F'(x)\right)^2}\,dx$

$= \int_1^2 \sqrt{1+16x^2+8x}\,dx$

$= \int_1^2 4x+1\,dx$

$= 2x^2+x\Big|_1^2 = 7$

1992 BC3 Part (c)

(c) distance is

$\int_0^1 \sqrt{\left(e^t \sin t + e^t \cos t\right)^2 + \left(e^t \cos t - e^t \sin t\right)^2}\,dt$

$= \int_0^1 \sqrt{2e^{2t}\left(\sin^2 t + \cos^2 t\right)}\,dt = \int_0^1 \sqrt{2}e^t\,dt$

$= \sqrt{2}\,e^t\Big|_0^1 = \sqrt{2}\,(e-1)$

1993 BC2 Part (b)

(b) $\int_0^5 \sqrt{4t^2+4t^4}\,dt$

$= \int_0^5 2t\sqrt{1+t^2}\,dt$

$= \frac{2}{3}\left(1+t^2\right)^{3/2}\Big|_0^5$

$= \frac{2}{3}\left(26^{3/2}-1\right)$

▦ **2002B BC1 Part (d)**

(d) Distance $= \int_{-\pi}^{\pi} \sqrt{9\cos^2(3t) + 4}\,dt$

$= 17.973 > 5\pi$