DATE: _____

Unit 9 (Chapter 10): Limits **Pre-Calculus 2016-2017**

10.3 More on Limits

Target 9A: Evaluate a limit of a function algebraically
Target 9C: Evaluate a limit of a function graphically

Review of Prior Concepts

HINT: Think about the parent functions

1. $\lim_{x\to\infty}(x^2+3x-2) = \infty$

2. $\lim_{x\to-\infty}(x^2+3x-2) = \infty$

3. $\lim_{x\to\infty}(x^3+2) = \infty$

4. $\lim_{x\to-\infty}(x^3+2) = -\infty$

5. $\lim_{x\to\infty}(e^x) = \infty$

6. $\lim_{x\to-\infty}(e^x) = 0$

More Practice

End Behavior

http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/14-tail-behavior-limits-at-infinity-02
https://quizlet.com/48500929/end-behavior-of-12-basic-functions-flash-cards/
http://www.mathguide.com/cgi-bin/quizmasters3/EB.cgi
https://youtu.be/Krjd_vU4Uvg

Limits at Infinity

(end behavior of a graph/function)

Example 1:

a) Find $\lim_{x\to\infty}\left(\frac{3}{x}\right) = 0^+$

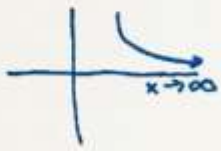
positive zero (zero from positive #s)

$\frac{3}{\infty}$

Some big #

$\frac{3}{100000000} = +.000000003$

As x approaches ∞, the graph of $\frac{3}{x}$ approaches 0.



graph goes to zero from positive y-values

b) Find $\lim_{x\to-\infty}\left(\frac{3}{x}\right) = 0^-$

negative zero (zero from negative #s)

$\frac{3}{-\infty}$

Some big neg #

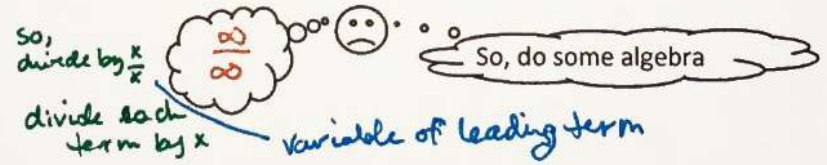
$\frac{3}{-10000000} = -.0000003$

As x approaches $-\infty$, the graph of $\frac{3}{x}$ approaches 0

graph goes to zero from negative y-values

Example 2:

a) Find $\lim_{x\to\infty}\left(\frac{x-2}{3x-4}\right)$



$$= \lim_{x\to\infty} \frac{\frac{x}{x} - \frac{2}{x}}{\frac{3x}{x} - \frac{4}{x}}$$

$$= \lim_{x\to\infty} \frac{1 - \frac{2}{x}}{3 - \frac{4}{x}}$$

$\frac{2}{x} \to \frac{2}{\infty} = 0$

$\frac{4}{x} \to \frac{4}{\infty} = 0$

$$= \boxed{\frac{1}{3}}$$

Check answer graphically (graph the function & the answer)

(Notice: as $x \to \infty$, graph approaches Horizontal Asymptote @ $y = \frac{1}{3}$)

b) Find $\lim_{x\to-\infty}\left(\frac{6x}{3x+4}\right)$

$$= \lim_{x\to-\infty} \frac{\frac{6x}{x}}{\frac{3x}{x} + \frac{4}{x}}$$

$$= \lim_{x\to-\infty} \frac{6}{3 + \frac{4}{x}}$$

$\frac{4}{x}$ goes to zero

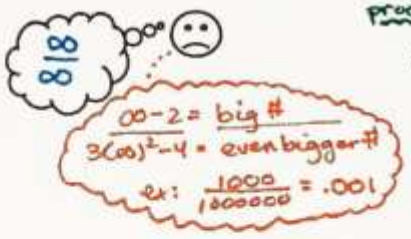
$$= \frac{6}{3} = \boxed{2}$$

c) Find $\lim_{x\to\infty}\left(\frac{8x-1}{2x+3}\right) = \frac{8}{2} = \boxed{4}$ (Can you see the answer quickly—w/o work?)

Conclusion:
If degree of Numerator = degree of Denominator, then $\lim_{x\to\infty} f(x) =$ coefficients of Leading terms of Numerator/Denominator
(and, Horizontal Asymptote @ $y =$ coefficients (#)

Example 3:

a) Find $\lim_{x\to\infty}\left(\frac{x-2}{3x^2-4}\right) = 0$



Proof:

$$\lim_{x\to\infty} \frac{\frac{x}{x^2} - \frac{2}{x^2}}{\frac{3x^2}{x^2} - \frac{4}{x^2}}$$

divide by variable largest leading term

$$= \lim_{x\to\infty} \frac{\frac{1}{x} - \frac{2}{x^2}}{3 - \frac{4}{x^2}}$$

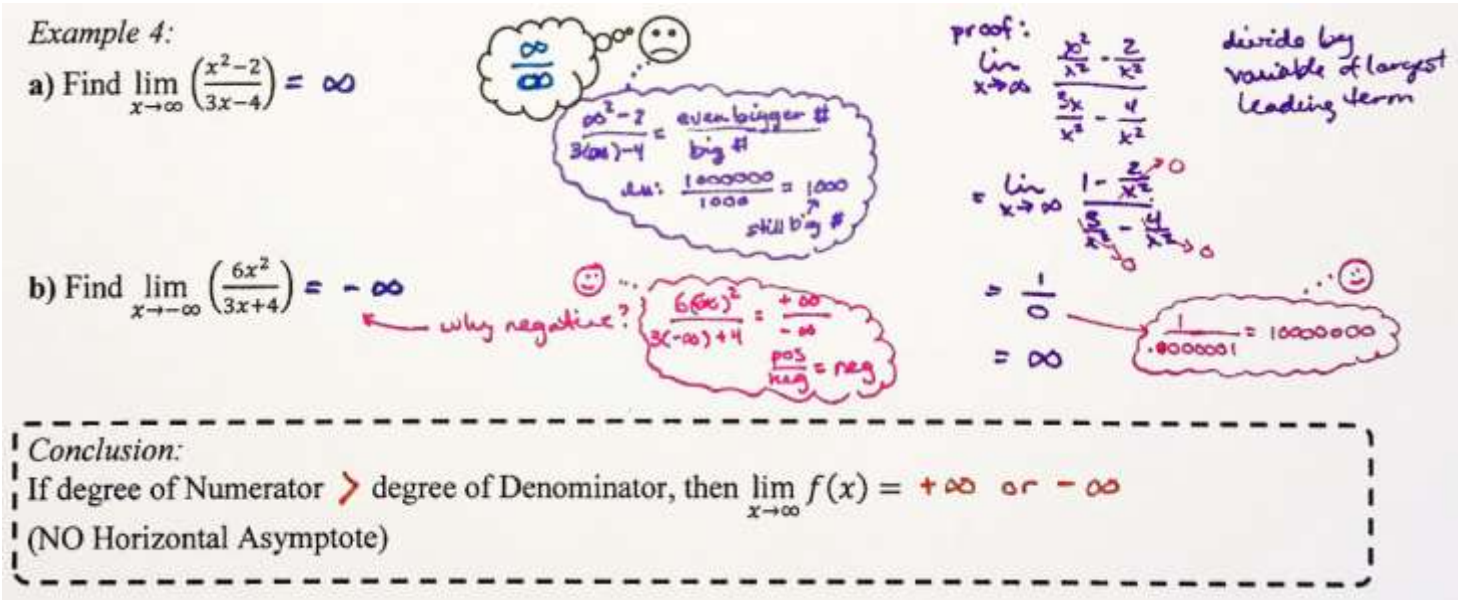
$\frac{1}{x} \to 0$, $\frac{2}{x^2} \to 0$, $\frac{4}{x^2} \to 0$

$$= \frac{0}{3}$$

$$= 0$$

b) Find $\lim_{x\to-\infty}\left(\frac{6x}{3x^2+4}\right) = 0$

Conclusion:
If degree of Numerator < degree of Denominator, then $\lim_{x\to\infty} f(x) = 0$
(and, Horizontal Asymptote @ $y = 0$



More Practice

Limits at Infinity

https://www.mathsisfun.com/calculus/limits-infinity.html
https://www.khanacademy.org/math/ap-calculus-ab/infinite-limits-ab/limits-at-infinity-ab/v/limits-and-infinity
http://www.shmoop.com/precalculus-limits/limits-infinity.html
https://youtu.be/wBYr-58mc5E
https://youtu.be/75xO9xy7TTQ
https://youtu.be/FVJNuukADeQ

Homework Assignment

Worksheet