

**10.3 More on Limits**

Target 9A: Evaluate a limit of a function algebraically  
Target 9D: Calculate one-sided limits and two-sided limits

*Review of Prior Concepts*

1.  $\lim_{x \rightarrow \infty} \left( \frac{x^2 - 4}{x + 2} \right)$

2.  $\lim_{x \rightarrow -\infty} \left( \frac{x^2 - 4}{x + 2} \right)$

3.  $\lim_{x \rightarrow 2} \left( \frac{x^2 - 4}{x + 2} \right)$

**More Practice****Limits at Infinity**

<https://www.mathsisfun.com/calculus/limits-infinity.html>

<https://www.khanacademy.org/math/ap-calculus-ab/infinite-limits-ab/limits-at-infinity-ab/v/limits-and-infinity>

<http://www.shmoop.com/precalculus-limits/limits-infinity.html>

<https://youtu.be/wBYr-58mc5E>

<https://youtu.be/75xO9xy7TTQ>

<https://youtu.be/FVJNuukADeQ>

**3 Methods for Evaluating Limits**

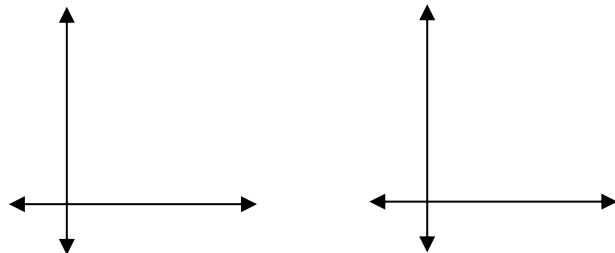
①

②

③

What does  $\lim_{x \rightarrow c} f(x) = L$  mean?

As  $x$  approaches  $c$  (from either side),  
then  $f(x)$  becomes close to  $L$ .



## Evaluate Limits Analytically/Algebraically

- Replace the value of  $c$  for  $x$  (if possible)

*Examples:*

1. Find  $\lim_{x \rightarrow 2} (x^3 - 2x)$

2. Find  $\lim_{x \rightarrow 1} 4x$

3. Find  $\lim_{x \rightarrow a} (x^3 + 4x)$

4. Find  $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x - 1}$

5. Find  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

6. Find  $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 9}$

7. Find  $\lim_{x \rightarrow 5} \sqrt{16 - x^2}$

8. Find  $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x - 1}$

Find: a)  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

b)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{2x}$

c)  $\lim_{x \rightarrow 0} \frac{\sin 10x}{10x}$

$$\lim_{x \rightarrow 0} \frac{\sin ax}{ax} = \quad , \text{ where } a \text{ is a constant } (\neq 0)$$

Examples:

1.  $\lim_{x \rightarrow 0} \frac{\sin 4x}{x}$

2.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$

3.  $\lim_{x \rightarrow 0} \frac{3x + \sin x}{x}$

4.  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^3 - 2x^2}$

### More Practice

#### Limits Analytically

<http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.4/>

<http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.5/>

<http://precalculus.flippedmath.com/151-limits-analytically.html>

<http://www.barrington220.org/cms/lib8/IL01001296/Centricity/Domain/321/1.3%20D1%20Ans.pdf>

<https://youtu.be/-gjURkNuh9o>

<https://youtu.be/MspCIN-r8C0>

### Homework Assignment

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