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Unit 9 (Chapter 10): Limits Pre-Calculus 2016-2017

10.3 More on Limits

Target 9A: Evaluate a limit of a function algebraically

Target 9D: Calculate one-sided limits and two-sided limits

Review of Prior Concepts

1. $\lim_{x\to\infty}\left(\frac{x^2-4}{x+2}\right) = \infty$

   b/c deg N > deg D, ∴, ∞

2. $\lim_{x\to-\infty}\left(\frac{x^2-4}{x+2}\right) = -\infty$

   deg N > deg D, ∴, ∞ or $-\infty$

   $\frac{(\infty)^2-4}{-\infty+2} = \frac{+\infty}{-\infty}$ → neg. #

3. $\lim_{x\to 2}\left(\frac{x^2-4}{x+2}\right) = \lim_{x\to 2}\frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

   $= \lim_{x\to 2}(x-2) = 0$

   factor + reduce

   check in graph calc

More Practice

Limits at Infinity

https://www.mathsisfun.com/calculus/limits-infinity.html

https://www.khanacademy.org/math/ap-calculus-ab/infinite-limits-ab/limits-at-infinity-ab/v/limits-and-infinity

http://www.shmoop.com/precalculus-limits/limits-infinity.html

https://youtu.be/wBYr-58mc5E

https://youtu.be/75xO9xy7TTQ

https://youtu.be/FVJNuukADeQ

3 Methods for Evaluating Limits

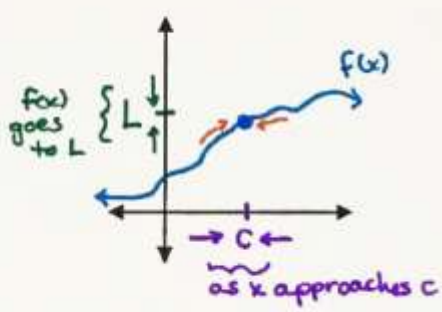
① Numerically – use table of values

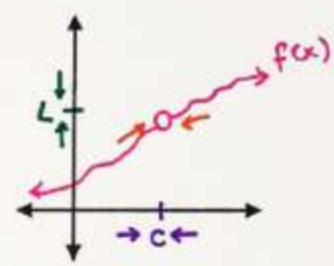
② Analytically – use algebra

③ Graphically – use graphs

What does $\lim_{x\to c} f(x) = L$ mean?

As x approaches c (from either side), then $f(x)$ becomes close to L.





Evaluate Limits Analytically/Algebraically

- Replace the value of c for x (if possible)

Examples:

1. Find $\lim_{x\to 2}(x^3 - 2x)$

$= 2^3 - 2(2)$

$= 8 - 4$

$= 4$

$\lim_{x\to 2}(x^3 - 2x) = \boxed{4}$

2. Find $\lim_{x\to 1} 4x$

$= 4(1)$

$= 4$

$\lim_{x\to 1} 4x = \boxed{4}$

3. Find $\lim_{x\to a}(x^3 + 4x)$

$= a^3 + 4a$

$\lim_{x\to a}(x^3 + 4x) = \boxed{a^3 + 4a}$

4. Find $\lim_{x\to -1} \frac{x^2-1}{x-1}$

$= \frac{(-1)^2 - 1}{-1-1}$

$= \frac{1-1}{-2}$

$= \frac{0}{-2} = \boxed{0}$

5. Find $\lim_{x\to 1} \frac{x^2-1}{x-1} = \frac{1^2-1}{1-1} = \frac{0}{0}$ ☹ ...do some algebra

$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$ Factor & reduce

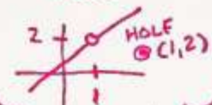
$= \lim_{x\to 1}(x+1)$

$= 1+1$

$= 2$

$\lim_{x\to 1} \frac{x^2-1}{x-1} = \boxed{2}$

☺ remember: removable discontinuity "hole"



6. Find $\lim_{x\to 3} \frac{x^2+2x-15}{x^2-9} = \frac{3^2+2(3)-15}{3^2-9} = \frac{0}{0}$ ☹

$= \lim_{x\to 3} \frac{(x+5)(x-3)}{(x+3)(x-3)}$

$= \lim_{x\to 3} \frac{(x+5)}{(x+3)}$

$= \frac{3+5}{3+3}$

$= \frac{8}{6}$

$= \boxed{\frac{4}{3}}$

7. Find $\lim_{x\to 5} \sqrt{16 - x^2}$

$= \sqrt{16-(5)^2}$

$= \sqrt{16-25}$

$= \sqrt{-9}$

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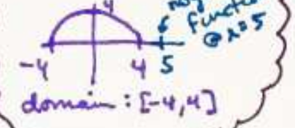
☺ $y = \sqrt{16-x^2}$

$y^2 = 16 - x^2$

$x^2 + y^2 = 16$

equation of circle w/ radius 4



domain: [-4,4]

8. Find $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \frac{\sqrt{1+3}-2}{1-1} = \frac{0}{0}$ ☹

$= \lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$ multiply by conjugate $\sqrt{a}+b \to \sqrt{a}-b$

$= \lim_{x\to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$

$= \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)}$ x-1 reduces ☺

$= \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2}$

$= \frac{1}{\sqrt{1+3}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$

Find: a) $\lim_{x\to 0}\frac{\sin x}{x} = 1$ b) $\lim_{x\to 0}\frac{\sin 2x}{2x} = 1$ c) $\lim_{x\to 0}\frac{\sin 10x}{10x} = 1$

$$\boxed{\lim_{x\to 0}\frac{\sin ax}{ax} = 1 \text{, where } a \text{ is a constant (\#)}}$$

Examples:

1. $\lim_{x\to 0}\frac{\sin 4x}{x}$ missing 4 by x

$= \lim_{x\to 0}\frac{4\cdot\sin 4x}{4\cdot x}$ multiply by $\frac{4}{4}$

$= \lim_{x\to 0} 4\cdot\frac{\sin 4x}{4x}$ now becomes 1

$= 4\cdot 1 = \boxed{4}$

2. $\lim_{x\to 0}\frac{\sin 3x}{2x}$ missing 3, extra 2

$= \lim_{x\to 0}\frac{3\cdot\sin 3x}{2\cdot 3\cdot x}$ multiply by $3/3$

$= \lim_{x\to 0}\frac{3}{2}\cdot\frac{\sin 3x}{3x}$ now becomes 1

$= \frac{3}{2}\cdot 1 = \boxed{\frac{3}{2}}$

3. $\lim_{x\to 0}\frac{3x+\sin x}{x}$ extra term, 3x

$= \lim_{x\to 0}\left(\frac{3x}{x}+\frac{\sin x}{x}\right)$ Split the fraction ex: $\frac{3+4}{5} \Rightarrow \frac{3}{5}+\frac{4}{5}$

$= \lim_{x\to 0}\frac{3x}{x} + \lim_{x\to 0}\frac{\sin x}{x}$

$= \lim_{x\to 0} 3 + \lim_{x\to 0}\frac{\sin x}{x}$ becomes 1

no place to put zero for x

$= 3 + 1$

$= \boxed{4}$

4. $\lim_{x\to 0}\frac{\sin^2 x}{x^3-2x^2}$ What a mess... Can't split a denominator

$= \lim_{x\to 0}\frac{(\sin x)(\sin x)}{x^2(x-2)}$ factor

$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{x}\cdot\frac{1}{x-2}$

$= \left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{x-2}\right)$ becomes 1 becomes 1

$= 1\cdot 1\cdot\frac{1}{0-2} = \boxed{-\frac{1}{2}}$

More Practice

Limits Analytically

http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.4/
http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.5/
http://precalculus.flippedmath.com/151-limits-analytically.html
http://www.barrington220.org/cms/lib8/IL01001296/Centricity/Domain/321/1.3%20D1%20Ans.pdf
https://youtu.be/-gjURkNuh9o
https://youtu.be/MspClN-r8C0

Homework Assignment

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