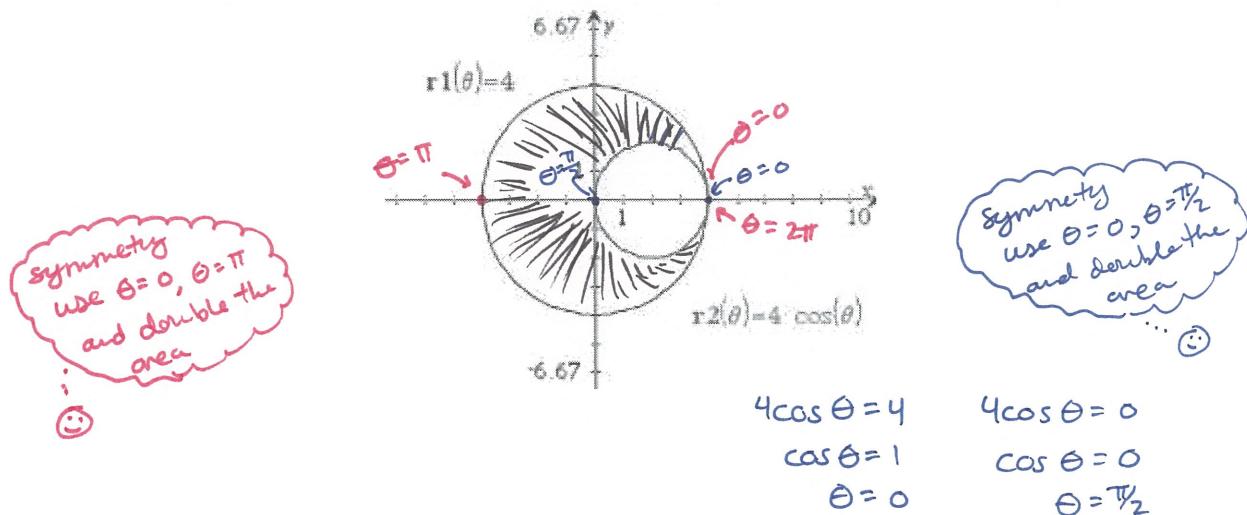


The graphs of the polar curves $r = 4$ and $r = 4 \cos \theta$ are shown in the figure below. Let R be the shaded region inside the graph of $r = 4 \cos \theta$, but outside the graph of $r = 4$. Find the area of R.



$$\begin{aligned}
 \text{Area of } R &= \frac{1}{2} \cdot 2 \int_0^{\pi} (4)^2 d\theta - \frac{1}{2} \cdot 2 \int_0^{\pi/2} (4 \cos \theta)^2 d\theta \\
 &= \int_0^{\pi} 16 d\theta - \int_0^{\pi/2} 16 \cos^2 \theta d\theta \\
 &= 16\theta \Big|_0^{\pi} - 16 \int_0^{\pi/2} \left(\frac{1}{2} \cos 2\theta + \frac{1}{2} \right) d\theta \\
 &= 16\pi - 16 \left(\frac{1}{4} \sin 2\theta + \frac{1}{2}\theta \right) \Big|_0^{\pi/2} \\
 &= 16\pi - 16 \left(\frac{1}{4} \sin \pi + \frac{\pi}{4} - 0 \right) \\
 &= 16\pi - 16 \left(\frac{\pi}{4} \right) \\
 &= 16\pi - 4\pi \\
 &= \boxed{12\pi}
 \end{aligned}$$

$$\begin{aligned}
 \cos 2\theta &= 2\cos^2 \theta - 1 \\
 \cos 2\theta + 1 &= 2\cos^2 \theta \\
 \frac{1}{2}\cos 2\theta + \frac{1}{2} &= \cos^2 \theta
 \end{aligned}$$

∴ Pre-Calc

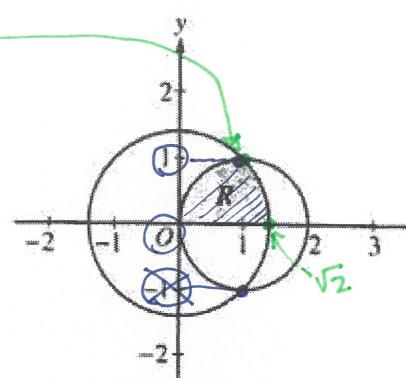
**AP[®] CALCULUS BC
2003 SCORING GUIDELINES (Form B)**

Question 2

The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x - 1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$.

Let R be the shaded region in the first quadrant bounded by the two circles and the x -axis.

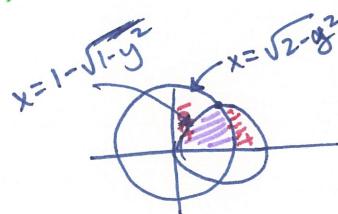
- Set up an expression involving one or more integrals with respect to x that represents the area of R .
- Set up an expression involving one or more integrals with respect to y that represents the area of R .
- The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ that represents the area of R .



a) $y = \sqrt{2-x^2}$ $y = \sqrt{1-(x-1)^2}$ $x^2 + y^2 = 2 \rightarrow y = \pm\sqrt{2-x^2}$
 $(x-1)^2 + y^2 = 1 \rightarrow y = \pm\sqrt{1-(x-1)^2}$

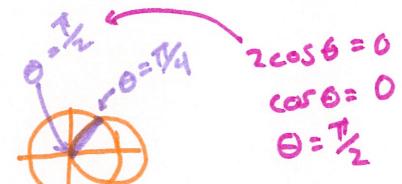
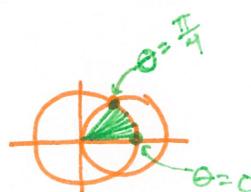
Area of $R = \int_0^1 \sqrt{1-(x-1)^2} dx + \int_1^{\sqrt{2}} \sqrt{2-x^2} dx$

b) $x^2 + y^2 = 2 \rightarrow x = \pm\sqrt{2-y^2}$
 $(x-1)^2 + y^2 = 1 \rightarrow x = 1 \pm\sqrt{1-y^2}$



Area of $R = \int_0^1 (\sqrt{2-y^2} - (1 - \sqrt{1-y^2})) dy$

c) $r = \sqrt{2}$
 $r = 2\cos\theta$ }
 $\frac{\sqrt{2}}{2} = \cos\theta$
 $\frac{\pi}{4} = \theta$



Area of $R = \frac{1}{2} \int_0^{\pi/4} (\sqrt{2})^2 d\theta + \frac{1}{2} \int_{\pi/4}^{\pi/2} (2\cos\theta)^2 d\theta$

Area of Polar Graphs: AP M/C Practice

1. Determine the area of the inner loop of the polar curve $r = 1 - 2\sin(\theta)$.

(A) 0.544

(B) 0.585

(C) 0.598

(D) 0.623

(E) 0.648

$$1 - 2\sin \theta = 0$$

$$1 = 2\sin \theta$$

$$\frac{1}{2} = \sin \theta$$

$$\frac{\pi}{6}, \frac{5\pi}{6} = \theta$$

$$\text{Area} = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 - 2\sin \theta)^2 d\theta$$

$$= 0.544$$

2. The graphs of the polar curves $r = 1$ and $r = 1 + \cos(\theta)$ are shown in the figure below.

If R is the region that is inside the graph of $r = 1$ and outside of the graph of $r = 1 + \cos(\theta)$, the area of R is:

(A) 1.127

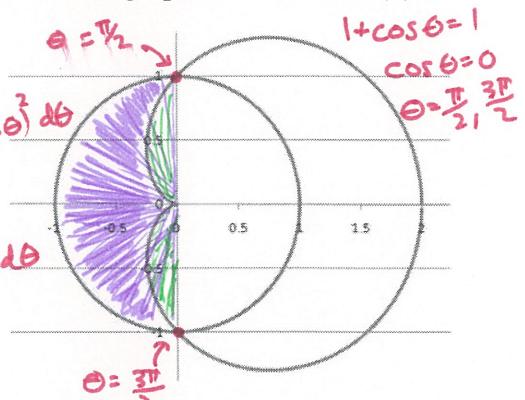
(B) 1.215

(C) 1.275

(D) 1.235

(E) 1.375

$$\begin{aligned} \text{Area of } R &= \frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1^2 d\theta - \frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1 + \cos \theta)^2 d\theta \\ &\text{OR} \\ &= \frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1^2 - (1 + \cos \theta)^2) d\theta \\ &= 1.215 \end{aligned}$$



3. The area of the closed region bounded by the polar graph of $r = \sqrt{3 + \cos \theta}$ is given by the integral:

- (A) $\int_0^{2\pi} \sqrt{3 + \cos \theta} d\theta$ *need $\frac{1}{2} \int r^2$*
- (B) $\int_0^{\pi} \sqrt{3 + \cos \theta} d\theta$
- (C) $2 \int_0^{\pi/2} (3 + \cos \theta) d\theta$
- (D) $\int_0^{\pi} (3 + \cos \theta) d\theta$
- (E) $2 \int_0^{\pi/2} \sqrt{3 + \cos \theta} d\theta$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \int_0^{2\pi} (\sqrt{3 + \cos \theta})^2 d\theta \\ &= \frac{1}{2} \int_0^{2\pi} (3 + \cos \theta) d\theta \\ &= \frac{1}{2} \cdot 2 \int_0^{\pi} (3 + \cos \theta) d\theta \\ &= \int_0^{\pi} (3 + \cos \theta) d\theta \end{aligned}$$

$$\begin{aligned} \sqrt{3 + \cos \theta} &= \sqrt{3} \\ \cos \theta &= 0 \\ \theta &= 0, 2\pi \end{aligned}$$

4. The area of the region enclosed by the polar curve $r = 1 - \cos \theta$ is

(A) $\frac{3\pi}{4}$ (B) π (C) $\frac{3\pi}{2}$ (D) 2π (E) 3π

$$\begin{aligned} \text{Area} &= \frac{1}{2} \int_0^{2\pi} (1 - \cos \theta)^2 d\theta \\ &= \frac{1}{2} \int_0^{2\pi} (1 - 2\cos \theta + \cos^2 \theta) d\theta \\ &= \frac{1}{2} \int_0^{2\pi} (1 - 2\cos \theta + \frac{1}{2}\cos 2\theta + \frac{1}{2}) d\theta \\ &= \frac{1}{2} (\theta - 2\sin \theta + \frac{1}{4}\sin 2\theta + \frac{1}{2}\theta) \Big|_0^{2\pi} \\ &= \frac{1}{2} (2\pi - 0 + 0 + \pi - 0) = \frac{1}{2}(3\pi) = \boxed{\frac{3\pi}{2}} \end{aligned}$$

$$\begin{aligned} 1 - \cos \theta &= 0 \\ 1 &= \cos \theta \\ 0, 2\pi &= \theta \end{aligned}$$