

**Unit 9 (Chapter 10): Limits****10.3 More on Limits**

Target 9C: Evaluate a limit of a function graphically

Target 9D: Calculate one-sided limits and two-sided limits

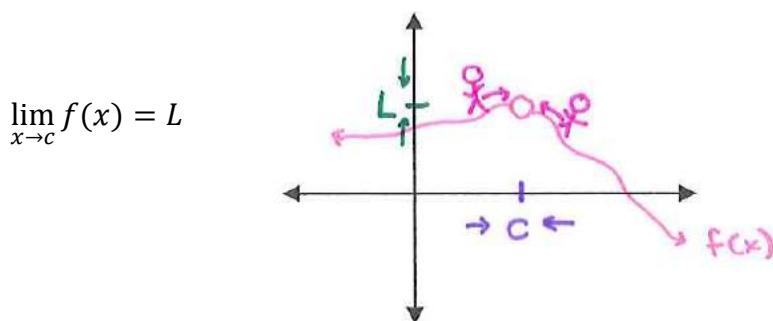
*Review of Prior Concepts*Given  $\lim_{x \rightarrow 3} f(x) = 2$  and  $\lim_{x \rightarrow 3} g(x) = -1$ , find:

a)  $\lim_{x \rightarrow 3} (2f(x) + g(x))$

$$\begin{aligned}
 &= 2 \underset{2}{\cancel{\lim_{x \rightarrow 3} f(x)}} + \underset{-1}{\cancel{\lim_{x \rightarrow 3} g(x)}} \\
 &= 2 \cdot 2 + -1 \\
 &= \boxed{3}
 \end{aligned}$$

b)  $\lim_{x \rightarrow 3} \frac{(g(x))^2}{f(x)-4}$

$$\begin{aligned}
 &= \left[ \underset{-1}{\cancel{\lim_{x \rightarrow 3} g(x)}} \right]^2 = \frac{(-1)^2}{2-4} = \boxed{\frac{1}{-2}}
 \end{aligned}$$

**More Practice****Limits Analytically**<http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.4/><http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.5/><http://precalculus.flippedmath.com/151-limits-analytically.html><http://www.barrington220.org/cms/lib8/IL01001296/Centricity/Domain/321/1.3%20D1%20Ans.pdf><https://youtu.be/-gjURkNuh9o><https://youtu.be/MspCIN-r8C0>**Finding Limits Graphically**In order for  $\lim_{x \rightarrow c} f(x)$  to exist,

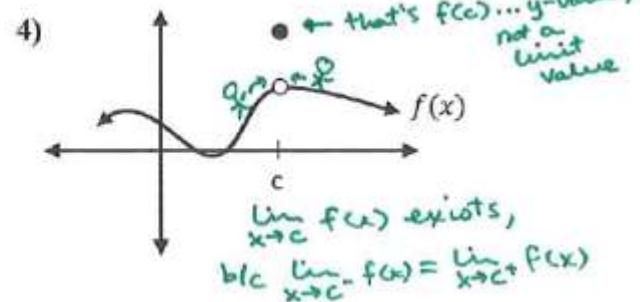
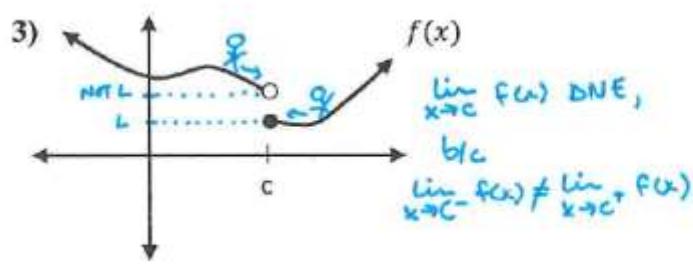
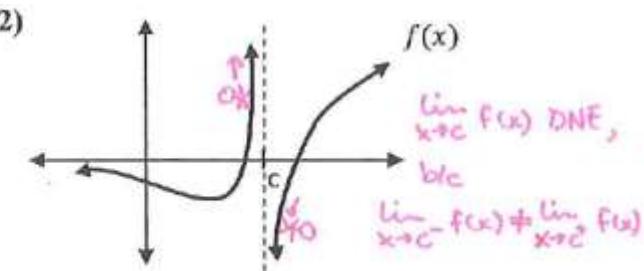
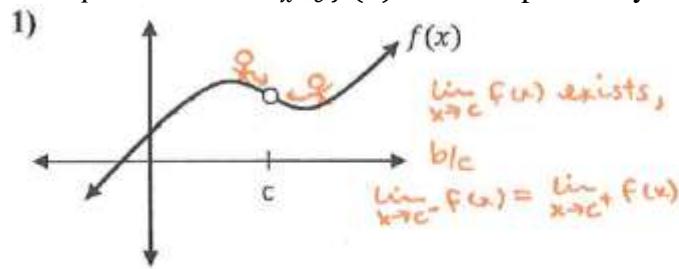
$\lim_{x \rightarrow c^-} f(x)$  has to equal  $\lim_{x \rightarrow c^+} f(x)$

*from left*                                   *from right*

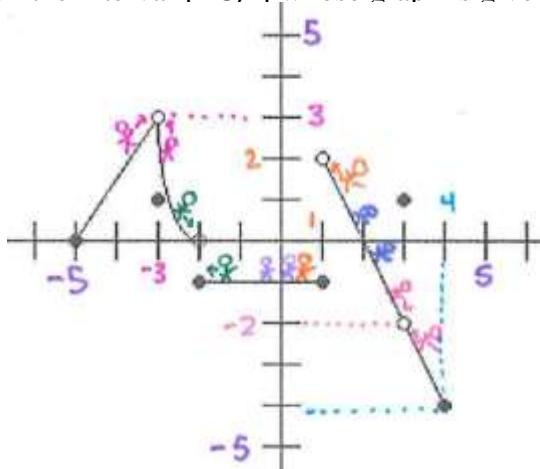
"limit from the left = limit from the right"

.. . ☺

Examples: Does  $\lim_{x \rightarrow c} f(x)$  exist? Explain why or why not.



Let  $g$  be a function defined on the interval  $[-5, 4]$  whose graph is given as:



Using the graph, find the following limits if they exist, and, if not, explain why not.

1.)  $\lim_{x \rightarrow 3} g(x) = -2$

6.)  $\lim_{x \rightarrow 1} g(x) = \text{DNE}$   
b/c  $\lim_{x \rightarrow 1^-} g(x) \neq \lim_{x \rightarrow 1^+} g(x)$

2.)  $\lim_{x \rightarrow 0} g(x) = -1$

7.)  $\lim_{x \rightarrow 2^-} g(x) = 0$

3.)  $\lim_{x \rightarrow -3} g(x) = 3$

8.)  $\lim_{x \rightarrow 4} g(x) = \text{DNE}$   
 $\lim_{x \rightarrow 4^-} g(x) = -4$   
 $\lim_{x \rightarrow 4^+} g(x) = \text{DNE}$   
 $\therefore \lim_{x \rightarrow 4^-} g(x) \neq \lim_{x \rightarrow 4^+} g(x)$

4.)  $\lim_{x \rightarrow 1^+} g(x) = 2$

9.)  $\lim_{x \rightarrow 2} g(x) = 0$

5.)  $\lim_{x \rightarrow 1^-} g(x) = -1$

10.)  $\lim_{x \rightarrow -2^+} g(x) = -1$

### More Practice

#### Limits Graphically

<http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/21-rational-functions-limits-infinity-right-left-01>

<https://www.khanacademy.org/math/ap-calculus-ab/limits-basics-ab/limits-from-graphs-ab/v/limits-from-graphs-undefined>

<http://precalculus.flippedmath.com/23-limits-graphically.html>

<http://philschatz.com/precalculus-book/contents/m49452.html>

<http://www.mathsisfun.com/calculus/limits.html>

<https://youtu.be/IklVRSonyFQ>

<https://youtu.be/UkjgJQaGx98>

[https://youtu.be/XOu\\_LFGai0A](https://youtu.be/XOu_LFGai0A)

### Homework Assignment

p.826 #27–33odd,37–45odd