DATE: _____

10.3 Polar Derivatives Practice Problems

Step-By-Step Multiple-Choice

Q14: Consider the polar equation $r = 2\sin\theta$. We can calculate the derivative $\frac{dy}{dx}$ by dividing the derivative $\frac{dy}{d\theta}$ by the derivative $\frac{dx}{d\theta}$.

- To calculate the derivative $\frac{dy}{d\theta}$, we first need to introduce the variable y by multiplying both sides of the equation by $\sin\theta$ and then substituting. Write this equation y in terms of θ.

  A $y = 2\sin 2\theta$

  B $y = 2\sin\theta$

  C $y = 4\sin^2\theta$

  D $y = 2\sin^2\theta$

  E $y = 2\sin\theta^2$

- Calculate the derivative $\frac{dy}{d\theta}$.

  A $\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

  B $\frac{dy}{d\theta} = 4\sin\theta$

  C $\frac{dy}{d\theta} = 8\sin\theta\cos\theta$

  D $\frac{dy}{d\theta} = 4\cos 2\theta$

  E $\frac{dy}{d\theta} = -4\sin\theta\cos\theta$

- Similarly, to calculate the derivative $\frac{dx}{d\theta}$, we first need to introduce the variable x by multiplying both sides of the original equation by $\cos\theta$ and then substituting. Write this equation x in terms of θ.

  A $x = y\cos\theta$

  B $x = 2\cos\theta$

  C $x = 2\sin\theta$

  D $x = 2\sin\theta\cos\theta$

  E $x = -y\cot\theta$

- Calculate the derivative $\frac{dx}{d\theta}$.

  A $\frac{dx}{d\theta} = 2\left(\cos^2\theta + \sin^2\theta\right)$

  B $\frac{dx}{d\theta} = \left(\cos^2\theta + \sin^2\theta\right)$

  C $x = 2\cos\theta$

  D $\frac{dx}{d\theta} = \cos 2\theta$

  E $\frac{dx}{d\theta} = 2\cos 2\theta$

The derivative $\frac{dy}{dx}$ is equal to $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$. Calculate $\frac{dy}{dx}$.

A $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{2\left(\cos^2\theta + \sin^2\theta\right)}$

B $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{\cos 2\theta}$

C $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{2\cos 2\theta}$

D $\frac{dy}{dx} = \frac{-4\sin\theta\cos\theta}{2\cos 2\theta}$

E $\frac{dy}{dx} = \frac{-4\sin\theta\cos\theta}{\cos 2\theta}$

Use the derivative function to calculate the slope of the tangent to $r = 2\sin\theta$ at $\theta = \frac{\pi}{6}$.

A $\frac{\sqrt{3}}{3}$

B $-\sqrt{3}$

C $\sqrt{3}$

D $2\sqrt{3}$

E $-2\sqrt{3}$

FRQs (Free-Response Questions)

1. Find the point(s) on the curve, $r = 4\cos\theta$, for $0 \le \theta < \pi$ where $\frac{dy}{dx} = 0$.

2. Find the slope of the tangent line to the graph of r, where $r = 2\theta$, in terms of θ.
Find the polar coordinates, $0 \le \theta < 2\pi$ where the curve has a vertical tangent line.

3. Find the tangent line for the polar curve $r = \theta \cos\theta$ at $\theta = 0$.

Multiple-Choice

4. Find the slope of the tangent line to the curve $r = \frac{1}{\theta}$ at $\theta = \pi$.

(A) $-\frac{1}{\pi}$

(B) $-\pi$

(C) 0

(D) π

(E) $\frac{1}{\pi}$

5. Find the slope of the tangent line to the curve $r = \cos\theta$ at $\theta = \frac{\pi}{6}$.

(A) $-\frac{\sqrt{3}}{3}$

(B) $-\frac{\sqrt{3}}{4}$

(C) $-\sqrt{3}$

(D) $\frac{\sqrt{3}}{3}$

(E) $\sqrt{3}$