DATE: ______________

10.3 Polar Derivatives Practice Problems

Step-By-Step Multiple-Choice

Q14: Consider the polar equation $r = 2\sin\theta$. We can calculate the derivative $\frac{dy}{dx}$ by dividing the derivative $\frac{dy}{d\theta}$ by the derivative $\frac{dx}{d\theta}$.

- To calculate the derivative $\frac{dy}{d\theta}$, we first need to introduce the variable y by multiplying both sides of the equation by $\sin\theta$ and then substituting. Write this equation y in terms of θ.

A $y = 2\sin 2\theta$

B $y = 2\sin\theta$

C $y = 4\sin^2\theta$

D $y = 2\sin^2\theta$

E $y = 2\sin\theta^2$

$y = r\sin\theta$
$y = 2\sin\theta\sin\theta$
$y = 2\sin^2\theta$

- Calculate the derivative $\frac{dy}{d\theta}$.

A $\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

B $\frac{dy}{d\theta} = 4\sin\theta$

C $\frac{dy}{d\theta} = 8\sin\theta\cos\theta$

D $\frac{dy}{d\theta} = 4\cos 2\theta$

E $\frac{dy}{d\theta} = -4\sin\theta\cos\theta$

$\frac{dy}{d\theta} = \sin\theta(2\cos\theta) + 2\sin\theta\cos\theta$
$= 2\sin\theta\cos\theta + 2\sin\theta\cos\theta$
$\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

- Similarly, to calculate the derivative $\frac{dx}{d\theta}$, we first need to introduce the variable x by multiplying both sides of the original equation by $\cos\theta$ and then substituting. Write this equation x in terms of θ.

A $x = y\cos\theta$

B $x = 2\cos\theta$

C $x = 2\sin\theta$

D $x = 2\sin\theta\cos\theta$

E $x = -y\cot\theta$

$x = r\cos\theta$
$x = 2\sin\theta\cos\theta$

- Calculate the derivative $\frac{dx}{d\theta}$.

A $\frac{dx}{d\theta} = 2\left(\cos^2\theta + \sin^2\theta\right)$

B $\frac{dx}{d\theta} = \left(\cos^2\theta + \sin^2\theta\right)$

C $x = 2\cos\theta$

D $\frac{dx}{d\theta} = \cos 2\theta$

E $\frac{dx}{d\theta} = 2\cos 2\theta$

$\frac{dx}{d\theta} = \cos\theta(2\cos\theta) + 2\sin\theta(-\sin\theta)$
$= 2\cos^2\theta - 2\sin^2\theta$
$= 2(\cos^2\theta - \sin^2\theta)$
$\frac{dx}{dt} = 2\cos 2\theta$

The derivative $\frac{dy}{dx}$ is equal to $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$. Calculate $\frac{dy}{dx}$.

A $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{2\left(\cos^2\theta + \sin^2\theta\right)}$

B $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{\cos 2\theta}$

C $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{2\cos 2\theta}$

D $\frac{dy}{dx} = \frac{-4\sin\theta\cos\theta}{2\cos 2\theta}$

E $\frac{dy}{dx} = \frac{-4\sin\theta\cos\theta}{\cos 2\theta}$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{4\cos\theta\sin\theta}{2\cos 2\theta}$

or $\frac{2\cos\theta\sin\theta}{\cos 2\theta}$

Use the derivative function to calculate the slope of the tangent to $r = 2\sin\theta$ at $\theta = \frac{\pi}{6}$.

A $\frac{\sqrt{3}}{3}$

B $-\sqrt{3}$

C $\sqrt{3}$

D $2\sqrt{3}$

E $-2\sqrt{3}$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = \frac{2\cos\pi/6 \sin\pi/6}{\cos(2\cdot\pi/6)}$

$= \frac{2(\sqrt{3}/2)(1/2)}{\cos \pi/3}$

$= \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}$

FRQs (Free-Response Questions)

1. Find the point(s) on the curve, $r = 4\cos\theta$, for $0 \le \theta < \pi$ where $\frac{dy}{dx} = 0$.

$x = 4\cos\theta\cos\theta$ $\quad y = 4\cos\theta\sin\theta$

$\to \frac{dy}{dx} = 0$ only when $dy/d\theta = 0$ and $dx/d\theta \neq 0$

$\frac{dy}{d\theta} = 4(\sin\theta(-\sin\theta) + \cos\theta\cos\theta)$

$0 = 4(-\sin^2\theta + \cos^2\theta)$

$0 = 4\cos 2\theta$

$0 = \cos 2\theta$

$2\theta = \pi/2, 3\pi/2 \to \boxed{\theta = \pi/4, 3\pi/4}$

$\frac{dx}{d\theta} = 4(\cos\theta(-\sin\theta) + \cos\theta(-\sin\theta))$

$= -8\cos\theta\sin\theta$

$\left.\frac{dx}{d\theta}\right|_{\theta=\pi/4} \neq 0$, $\left.\frac{dx}{d\theta}\right|_{\theta=3\pi/4} \neq 0$

2. Find the slope of the tangent line to the graph of r, where $r = 2\theta$, in terms of θ. Find the polar coordinates, $0 \le \theta < 2\pi$ where the curve has a vertical tangent line.

$x = 2\theta\cos\theta$ $\quad y = 2\theta\sin\theta$

$\frac{dy}{dx} = \frac{\sin\theta(2) + 2\theta(\cos\theta)}{\cos\theta(2) + 2\theta(-\sin\theta)}$

$= \frac{2(\sin\theta + \theta\cos\theta)}{2(\cos\theta - \theta\sin\theta)}$

$= \frac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}$

$\frac{dx}{d\theta} = 2(\cos\theta - \theta\sin\theta)$

$0 = 2(\cos\theta - \theta\sin\theta)$

$\theta = 0.860, 3.426$

$\boxed{(1.721, 0.860) \text{ and } (6.851, 3.426)}$

3. Find the tangent line for the polar curve $r = \theta\cos\theta$ at $\theta = 0$.

$y - y_1 = m(x - x_1)$

$x = \theta\cos\theta\cos\theta$

$x(0) = 0$

$y = \theta\cos\theta\sin\theta$

$y(0) = 0$

$\boxed{y = 0}$

$$\frac{dy}{dx} = \frac{\sin\theta(\cos\theta + \theta(-\sin\theta)) + \theta\cos\theta(\cos\theta)}{\cos\theta(\cos\theta + \theta(-\sin\theta)) + \theta\cos\theta(-\sin\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{0(1+0) + 0(0)}{1(1+0) + 0} = \frac{0}{1} = 0$$

Multiple-Choice

4. Find the slope of the tangent line to the curve $r = \frac{1}{\theta}$ at $\theta = \pi$.

(A) $-\frac{1}{\pi}$

$\boxed{\textbf{(B)}}$ $-\pi$

(C) 0

(D) π

(E) $\frac{1}{\pi}$

$x = \frac{1}{\theta}\cos\theta = \frac{\cos\theta}{\theta}$ $\quad y = \frac{\sin\theta}{\theta}$

$$\frac{dy}{dx} = \frac{\frac{\theta\cos\theta - \sin\theta}{\theta^2}}{\frac{-\theta\sin\theta - \cos\theta}{\theta^2}} = \frac{\theta\cos\theta - \sin\theta}{\theta^2} \cdot \frac{\theta^2}{-\theta\sin\theta - \cos\theta}$$

$$= \frac{\theta\cos\theta - \sin\theta}{-\theta\sin\theta - \cos\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{\pi(-1) - 0}{-\pi(0) - (-1)} = \frac{-\pi}{1} = -\pi$$

5. Find the slope of the tangent line to the curve $r = \cos\theta$ at $\theta = \frac{\pi}{6}$.

(A) $-\frac{\sqrt{3}}{3}$

(B) $-\frac{\sqrt{3}}{4}$

(C) $-\sqrt{3}$

(D) $\frac{\sqrt{3}}{3}$

(E) $\sqrt{3}$

$x = \cos\theta\cos\theta$ $\quad y = \cos\theta\sin\theta$

$$\frac{dy}{dx} = \frac{\sin\theta(-\sin\theta) + \cos\theta(\cos\theta)}{\cos\theta(-\sin\theta) + \cos\theta(-\sin\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = \frac{(1/2)(-1/2) + (\sqrt{3}/2)(\sqrt{3}/2)}{(\sqrt{3}/2)(-1/2) + (\sqrt{3}/2)(-1/2)}$$

$$= \frac{-1/4 + 3/4}{-\sqrt{3}/4 - \sqrt{3}/4}$$

$$= \frac{1/2}{-\sqrt{3}/2} = \frac{1}{2} \cdot -\frac{2}{\sqrt{3}} = \frac{1}{-\sqrt{3}} = \frac{-\sqrt{3}}{3}$$