DATE: ____________

Power Series Theorems Practice

Express each function as a power series and determine the interval of convergence.

1. $f(x) = \frac{2}{(1+2x)^2}$

$\frac{d}{dx}\left(\frac{a}{1-r}\right)$ or $\int \frac{a}{1-r}$ ...

$\frac{d}{dx}\left(a(1-r)^{-1}\right) \overset{?}{=} 2(1+2x)^{-2}$

or $\int a(1-r)^{-1} \overset{?}{=} 2(1+2x)^{-2}$

$\int \frac{2}{(1+2x)^2}dx$ $\quad u = 1+2x$, $du = 2dx$

$\int \frac{1}{u^2}du$

$= -\frac{1}{u} + C$

$= -\frac{1}{1+2x} + C$

so, $\frac{d}{dx}\left(-\frac{1}{1+2x}\right) = \frac{2}{(1+2x)^2}$

$a = -1$
$r = -2x$

$$-\frac{1}{1+2x} = \sum_{n=0}^{\infty} (-1)(-2x)^n$$

$$= \sum_{n=0}^{\infty} (-1)(-2)^n x^n$$

$$\frac{d}{dx}\left(\frac{-1}{1+2x}\right) = \sum_{n=0}^{\infty} (-1)(-2)^n \cdot n \cdot x^{n-1}$$

$$\boxed{\frac{2}{(1+2x)^2} = \sum_{n=0}^{\infty} (-1)(-2)^n \cdot n \cdot x^{n-1}}$$

---

2. $g(x) = \frac{1}{2}\ln(x^2+1)$

$\frac{d}{dx}\left(\frac{a}{1-r}\right)$? or $\int \frac{a}{1-r}$? ...

$\frac{d}{dx}\left(\frac{a}{1-r}\right) \overset{?}{=} \frac{1}{2}\ln(x^2+1)$

or $\int \frac{a}{1-r} \overset{?}{=} \frac{1}{2}\ln(x^2+1)$

$\frac{d}{dx}\left(\frac{1}{2}\ln(x^2+1)\right)$

$= \frac{1}{2} \cdot 2x \cdot \frac{1}{x^2+1}$

$= \frac{x}{x^2+1}$

so, $\int \frac{x}{x^2+1}dx = \frac{1}{2}\ln(x^2+1)$

$a = x$
$r = -x^2$

$$\frac{x}{x^2+1} = \sum_{n=0}^{\infty} x \cdot (-x^2)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n+1}$$

$$\int \frac{x}{x^2+1}dx = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{2n+2} x^{2n+2}$$

$$\boxed{\frac{1}{2}\ln(x^2+1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+2} \cdot x^{2n+2}}$$

---

3. $h(x) = \tan^{-1}(4x)$

$\frac{d}{dx}\left(\frac{a}{1-r}\right)$? or $\int \frac{a}{1-r}$? ...

$\frac{d}{dx}\left(\frac{a}{1-r}\right) \overset{?}{=} \tan^{-1}(4x)$

or $\int \frac{a}{1-r} = \tan^{-1}(4x)$

$\frac{d}{dx}\left(\tan^{-1}(4x)\right)$

$= 4 \cdot \frac{1}{1+(4x)^2}$

$= \frac{4}{1+16x^2}$

so, $\int \frac{4}{1+16x^2}dx = \tan^{-1}(4x)$

$a = 4$
$r = -16x^2$

$$\frac{4}{1+16x^2} = \sum_{n=0}^{\infty} 4(-16x^2)^n$$

$$= \sum_{n=0}^{\infty} 4(-1)^n 16^n x^{2n}$$

$$= \sum_{n=0}^{\infty} 4 \cdot 4^{2n} (-1)^n x^{2n}$$

$$= \sum_{n=0}^{\infty} 4^{2n+1} (-1)^n x^{2n}$$

$$\int \frac{4}{1+16x^2}dx = \sum_{n=0}^{\infty} (-1)^n \cdot 4^{2n+1} \cdot \frac{1}{2n+1} x^{2n+1}$$

$$\boxed{\tan^{-1}(4x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} (4x)^{2n+1}}$$