

Evaluating Limits Analytically – Multiple Choice

1. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 4}$ is:

- (A) 1
 (B) 0
 (C) $-\frac{1}{2}$
 (D) -1
 (E) ∞

$$\begin{aligned} \lim_{k \rightarrow 2} \frac{x^2 - 4}{x^2 + 4} &= \frac{2^2 - 4}{2^2 + 4} \\ &= \frac{0}{8} \\ &= \boxed{0} \end{aligned}$$

2. $\lim_{x \rightarrow 3} \frac{x-3}{x^2 - 2x - 3}$ is:

- (A) 0
 (B) 1
 (C) $\frac{1}{4}$
 (D) ∞
 (E) none of these

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x-3}{x^2 - 2x - 3} &\rightarrow \frac{3-3}{3^2 - 2(3) - 3} = \frac{0}{0} \therefore \\ &= \lim_{x \rightarrow 3} \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)} \\ &= \lim_{x \rightarrow 3} \frac{1}{x+1} = \frac{1}{3+1} \\ &= \boxed{\frac{1}{4}} \end{aligned}$$

3. $\lim_{x \rightarrow 0} \frac{x}{x}$ is:

- (A) 1
 (B) 0
 (C) ∞
 (D) -1
 (E) nonexistent

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x}{x} &\rightarrow \frac{0}{0} \therefore \\ \lim_{x \rightarrow 0} \frac{\cancel{x}}{\cancel{x}} &= \lim_{x \rightarrow 0} 1 \\ &= \boxed{1} \end{aligned}$$

4. Evaluate the limit, if it exists: $\lim_{x \rightarrow 9} \frac{\sqrt{x-5} - 2}{x-9} \rightarrow \frac{\sqrt{9-5} - 2}{9-9} = \frac{0}{0} \therefore$

- (A) $\frac{1}{4}$
 (B) $-\frac{1}{4}$
 (C) 1
 (D) 0
 (E) The limit does not exist

$$\begin{aligned} \lim_{k \rightarrow 9} \frac{\sqrt{x-5} - 2}{x-9} \cdot \frac{\sqrt{x-5} + 2}{\sqrt{x-5} + 2} \\ &= \lim_{x \rightarrow 9} \frac{x-5-4}{(x-9)(\sqrt{x-5} + 2)} \\ &= \lim_{x \rightarrow 9} \frac{x-9}{(x-9)(\sqrt{x-5} + 2)} \\ &= \lim_{k \rightarrow 9} \frac{1}{\sqrt{x-5} + 2} = \frac{1}{\sqrt{9-5} + 2} = \frac{1}{2+2} = \boxed{\frac{1}{4}} \end{aligned}$$

5. Evaluate the limit: $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 3x} \rightarrow \frac{\sin 0}{0^2 + 3(0)} = \frac{0}{0} \therefore$

- (A) 1
 (B) $\frac{1}{3}$
 (C) 3
 (D) ∞
 (E) $\frac{1}{4}$

$$\begin{aligned} &= \lim_{k \rightarrow 0} \frac{\sin x}{x(x+3)} \\ &= \lim_{k \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{x+3} \right) \\ &= \lim_{k \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{k \rightarrow 0} \frac{1}{x+3} \\ &= 1 \cdot \frac{1}{0+3} \\ &= \boxed{\frac{1}{3}} \end{aligned}$$