



Given $h(x) = f(x) + g(x)$, $m(x) = f(g(x))$, and $w(x) = 3(g(x))^2 - f(x)$, using the graphs above, find each limit.

$$\begin{aligned}
 \text{a) } \lim_{x \rightarrow 0^+} h(x) &= \lim_{x \rightarrow 0^+} (f(x) + g(x)) \\
 &= \lim_{x \rightarrow 0^+} f(x) + \lim_{x \rightarrow 0^+} g(x) \\
 &= 0 + 2 \\
 &= \boxed{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \lim_{x \rightarrow 2^-} m(x) &= \lim_{x \rightarrow 2^-} f(g(x)) \\
 &= f\left(\lim_{x \rightarrow 2^-} g(x)\right) \\
 &= f(-2) \\
 &= \boxed{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \lim_{x \rightarrow 2^+} (m(x) + h(x)) &= \lim_{x \rightarrow 2^+} (f(g(x)) + f(x) + g(x)) \\
 &= \lim_{x \rightarrow 2^+} f(g(x)) + \lim_{x \rightarrow 2^+} f(x) + \lim_{x \rightarrow 2^+} g(x) \\
 &= f\left(\lim_{x \rightarrow 2^+} g(x)\right) + 8 + 0 \\
 &= f(0) + 8 \\
 &= 0 + 8 \\
 &= \boxed{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \lim_{x \rightarrow 4} w(x) &= \lim_{x \rightarrow 4} (3(g(x))^2 - f(x)) \\
 &= 3\left(\lim_{x \rightarrow 4} g(x)\right)^2 - \lim_{x \rightarrow 4} f(x) \\
 &= 3(4)^2 - 6 \\
 &= 48 - 6 \\
 &= \boxed{42}
 \end{aligned}$$

p.61 in your textbook gives the Properties of Limits.

Show your steps in applying these properties to the following problems:

Evaluate each limit, if: $\lim_{x \rightarrow 1} f(x) = 5$ and $\lim_{x \rightarrow 1} g(x) = -2$.

a) $\lim_{x \rightarrow 1} (f(x) + g(x))$

$$= \lim_{x \rightarrow 1} f(x) + \lim_{x \rightarrow 1} g(x)$$

$$= 5 + -2$$

$$= \boxed{3}$$

b) $\lim_{x \rightarrow 1} (2g(x) - 3f(x))$

$$= 2 \lim_{x \rightarrow 1} g(x) - 3 \lim_{x \rightarrow 1} f(x)$$

$$= 2(-2) - 3(5)$$

$$= -4 - 15$$

$$= \boxed{-19}$$

c) $\lim_{x \rightarrow 1} f(x)g(x)$

$$= \lim_{x \rightarrow 1} f(x) \cdot \lim_{x \rightarrow 1} g(x)$$

$$= 5 \cdot -2$$

$$= \boxed{-10}$$

d) $\lim_{x \rightarrow 1} \frac{f(x)+2}{f(x)g(x)}$

$$= \frac{\lim_{x \rightarrow 1} f(x) + \lim_{x \rightarrow 1} 2}{\lim_{x \rightarrow 1} f(x) \cdot \lim_{x \rightarrow 1} g(x)}$$

$$= \frac{5 + 2}{5(-2)}$$

$$= \frac{7}{-10}$$

$$= \boxed{\frac{7}{-10}}$$