

Continuity (Removable & Non-Removable) – Multiple Choice

1. On which of the following intervals is f continuous?

(A) $-1 \leq x \leq 0$

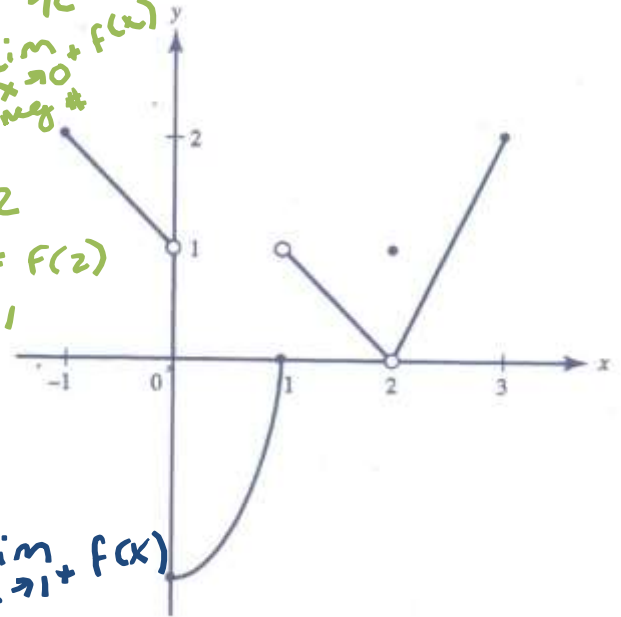
(B) $0 < x < 1$

(C) $1 \leq x \leq 2$

(D) $2 \leq x \leq 3$

(E) none of these

f discontinuous @ $x=1$ b/c $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$
 $\lim_{x \rightarrow 1^-} f(x) = 1$
 $\lim_{x \rightarrow 1^+} f(x) = 0$
 $1 \neq 0$
f discontinuous @ $x=2$ b/c $\lim_{x \rightarrow 2} f(x) \neq f(2)$
 $\lim_{x \rightarrow 2} f(x) = 1$
 $f(2) = 0$
 $1 \neq 0$



2. The function f has a jump discontinuity at

(A) $x = -1$

(B) $x = 1$

(C) $x = 2$

(D) $x = 3$

(E) none of these

b/c $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$
 $\lim_{x \rightarrow 1^-} f(x) = 1$
 $\lim_{x \rightarrow 1^+} f(x) = 0$
 $1 \neq 0$

3. The function f has a removable discontinuity at

(A) $x = 0$

(B) $x = 1$

(C) $x = 2$

(D) $x = 3$

(E) none of these

b/c $\lim_{x \rightarrow 2} f(x) \neq f(2)$
 $\lim_{x \rightarrow 2} f(x) = 1$
 $f(2) = 0$
 $1 \neq 0$

4. The graph of $y = \frac{x^2 - 9}{3x - 9}$ has

(A) a vertical asymptote at $x = 3$

(B) a horizontal asymptote at $y = \frac{1}{3}$

(C) a removable discontinuity at $x = 3$

(D) an infinite discontinuity at $x = 3$

(E) none of these

$$y = \frac{x^2 - 9}{3x - 9} = \frac{(x-3)(x+3)}{3(x-3)}$$

$x-3=0$
 $x=3$
 removable disc. @ $x=3$

5. The function

$$f(x) = \begin{cases} x^2 & x \neq 0 \\ 0 & x = 0 \end{cases} \xrightarrow{\text{reduce}} f(x) = \begin{cases} x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

- (A) is continuous everywhere
(B) is continuous except at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an infinite discontinuity at $x = 0$
(E) has $x = 0$ as a vertical asymptote

$$\begin{aligned} f(0) &= 0 \\ \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} x \\ &= 0 \\ f(0) &= \lim_{x \rightarrow 0} f(x) \quad \therefore f \text{ is cont @ } x=0 \end{aligned}$$

6. Suppose $\lim_{x \rightarrow -3^-} f(x) = -1$, $\lim_{x \rightarrow -3^+} f(x) = -1$, and $f(-3)$ is not defined.

Which of the following statements is (are) true?

TRUE I. $\lim_{x \rightarrow -3} f(x) = -1$ b/c $\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x) = -1$

FALSE II. f is continuous everywhere except at $x = -3$.
 maybe only given info about $x = -3$, not enough info about rest of $f(x)$

TRUE III. f has a removable discontinuity at $x = -3$.
 "hole"

- (A) None of them
(B) I only
(C) III only
(D) I and III only
(E) All of them

True, b/c $f(-3)$ not defined, but $\lim_{x \rightarrow -3} f(x) = -1$