

## Intermediate Value Theorem FRQ Practice

1. The rate at which water flows out of a pipe, in gallons per hour, is given by a continuous function  $R$  of time  $t$ . The table below shows the rate as measured every 3 hours for a 24-hour period. Is there some time  $t$ ,  $0 < t < 24$ , such that the rate of water flowing out of the pipe will be 11 gallons per hour? Justify your answer.

$t$ (hours)	0	3	6	9	12	15	18	21	24
$R(t)$ (gallons per hour)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

$R(t)$  is a continuous function b/c given in the problem

$$R(9) = 11.2 > 11$$

$$R(6) = 10.8 < 11$$

want  $R(t) = 11$  ☺  
need  $R(t) > 11$   
 $R(t) < 11$

∴, by IVT, since  $R(t)$  is cont and  $R(6) < 11 < R(9) > 11$ , then there is some time on  $(0, 24)$ , such that the rate of water flowing out of the pipe will be 11 gallons per hour.

2. Let  $f$  be a function that is continuous for all real numbers. The table below gives the values of  $f$  for selected points  $x$  in the closed interval  $-1.5 \leq x \leq 1.5$ . Find a value of  $r$  having the property that there must exist a value  $c$  with  $0 < c < 0.5$  and  $f(c) = r$ . Justify your answer.

$x$	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	7	5	4	1	3	5	8

↑  
need to find an  $r$ .

↕  
 $r$  b/n 1 and 3

$f$  is continuous for all real #s. b/c given in problem.

$$\text{Let } r = 2$$

☺  
 $r$  can be any value b/n 1 and 3

∴, by IVT, since  $f$  is cont and  $f(0.5) > 2$  and  $f(0) < 2$ ,

∃ a value of  $c$  on  $(0, 0.5)$  where  $f(c) = 2$ .

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table below gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where  $x$  represents the distance from one end of the blood vessel and  $B(x)$  is a continuous function that represents the diameter at that point. Based on the values in the table, what is the smallest number of instances at which the diameter of the blood vessel could be 25 mm? Justify your answer.

how many times will  $B(x) = 25$ ?

Distance $x$ (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

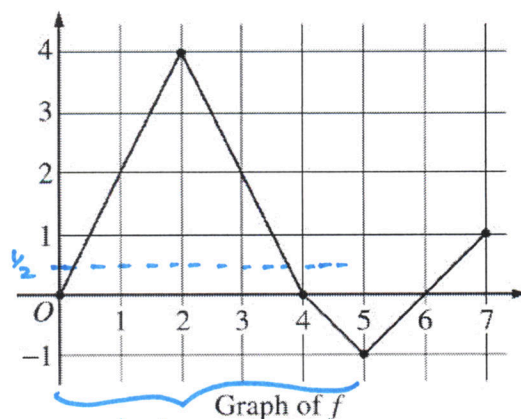
There are 3 instances in which the diameter of blood vessel could be 25 mm. b/c of IVT.

$B(x)$  is continuous b/c given in problem

$B(0) = 24 < 25$   
 $B(60) = 30 > 25$   
 $B(300) = 24 < 25$   
 $B(360) = 26 > 25$

} first instance,  $B(x) = 25$  on  $(0, 60)$   
 } second instance,  $B(x) = 25$  on  $(60, 300)$   
 } 3rd instance,  $B(x) = 25$  on  $(300, 360)$

4. Let  $f$  be a function defined on the closed interval  $[0, 7]$ . The graph of  $f$ , consisting of four line segments, is shown below. For how many values of  $c$ , where  $0 < c < 5$ , will  $f(x) = \frac{1}{2}$ ? Justify your answer.



look only on  $(0, 5)$   
 $f(x) = \frac{1}{2}$  on  $(0, 5)$  for 2 values of  $c$  b/c of IVT.

$f$  is continuous function as given in graph.

$f(0) = 0 < \frac{1}{2}$   
 $f(1) = 2 > \frac{1}{2}$

}  $\therefore f(x) = \frac{1}{2}$  on  $(0, 1)$

$f(3) = 2 > \frac{1}{2}$   
 $f(4) = 0 < \frac{1}{2}$

}  $\therefore f(x) = \frac{1}{2}$  on  $(3, 4)$

thus,  $f(x) = \frac{1}{2}$  for 2 values of  $c$  where  $0 < c < 5$